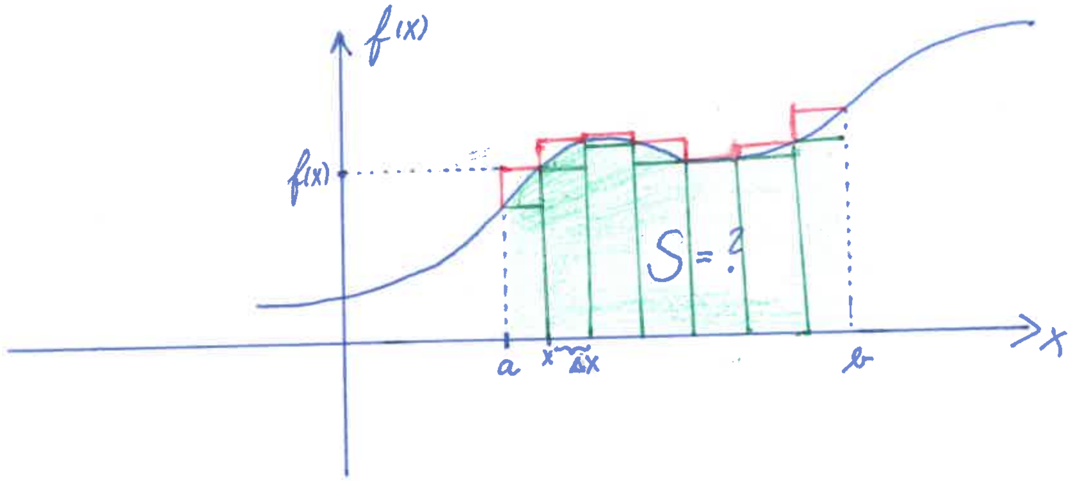
Určitý (Riemannův) integrál



Je-li funkce f spojitá a nezáporná na intervalu $\langle a, b\rangle$, kde $-\infty < a < b < +\infty$, potom obsahem plochy $\{(x,y) \in \mathbb{R}^2 \mid x \in \langle a,b\rangle, y \in \langle 0, f(x)\rangle\}$ je číslo

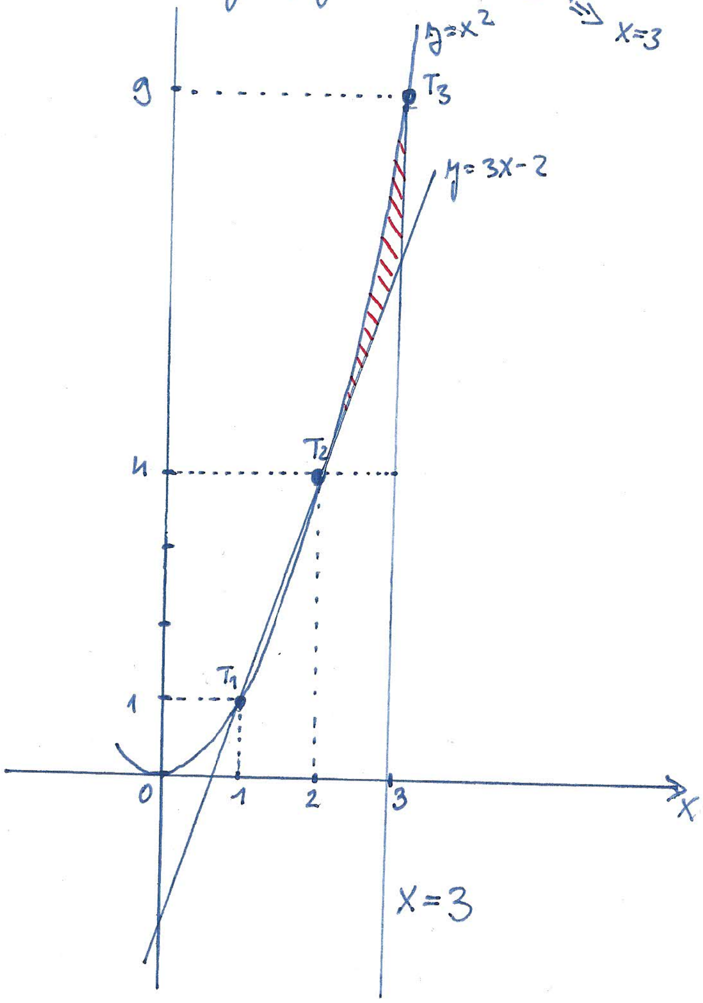
$$S = \int_a^b f(x)\,dx$$ .... Riemannův integrál funkce f od a do b

Věta: Jestliže f je v $\langle a, b\rangle$ spojitá nebo monotonní $\Rightarrow \int_a^b f(x)dx$ existuje.

Věta: Nechť f a F jsou spojité v $\langle a, b\rangle$, kde $-\infty < a < b < +\infty$ a $\forall x \in (a,b): F'(x) = f(x)$. Potom platí:

$$\int_a^b f(x)\,dx = F(b) - F(a) \overset{\text{ozn.}}{=} [F(x)]_a^b$$

① Vypočtěte obsah plochy ohraničené křivkami $y=x^2$, $y=3x-2$, $\underbrace{x-3=0}_{\Rightarrow x=3}$



$$S=\int_2^3\left(x^2-(3x-2)\right)dx=$$

$$=\int_2^3(x^2-3x+2)\,dx=$$

$$=\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_2^3=$$

$$=\left(\frac{27}{3}-\frac{27}{2}+6\right)-\left(\frac{8}{3}-\frac{12}{2}+4\right)=$$

$$=\frac{19}{3}-\frac{15}{2}+2=\frac{38-45}{6}+2=\frac{-7}{6}+\frac{12}{6}=\underline{\underline{\frac{5}{6}}}$$

VÝPOČET PRŮSEČÍKŮ:

a) $\underbrace{y=x^2=3x-2=y}$

$x^2-3x+2=0$

$(x-1)(x-2)=0$

$x_1=1 \quad x_2=2$

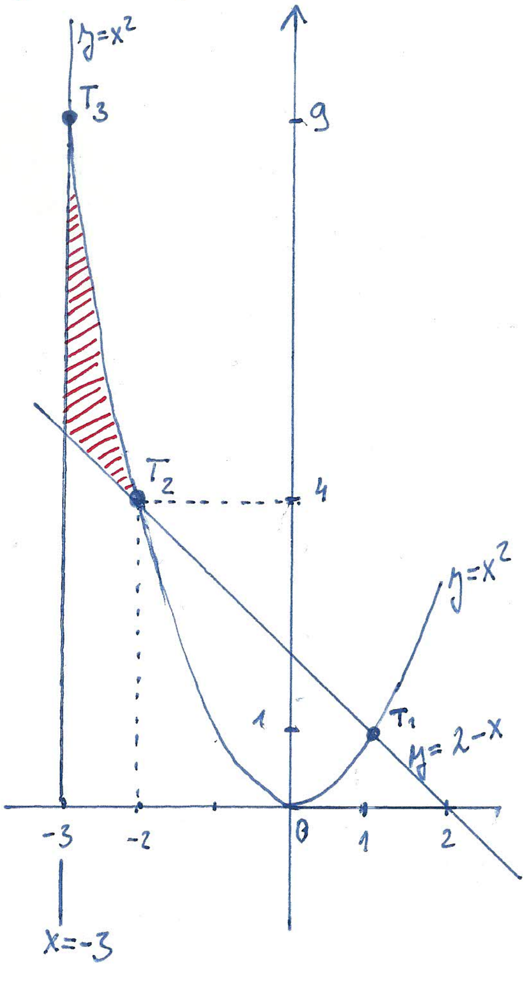
$T_1=[1,1]$; $T_2=[2,4]$

b) $y=x^2$ $\quad \underbrace{x-3=0}_{\Rightarrow x=3}$

$y=9$

$T_3=[3,9]$

② Vypočtěte obsah plochy ohraničené křivkami $y=2-x$, $y=x^2$, $x+3=0$.



$$S=\int_{-3}^{-2}\left(x^2-(2-x)\right)dx=$$

$$=\int_{-3}^{-2}(x^2+x-2)\,dx=\left[\frac{x^3}{3}+\frac{x^2}{2}-2x\right]_{-3}^{-2}=$$

$$=\left(-\frac{8}{3}+\frac{4}{2}+4\right)-\left(-\frac{27}{3}+\frac{9}{2}+6\right)=\frac{19}{3}-\frac{5}{2}-2=\frac{38-15-12}{6}=\underline{\underline{\frac{11}{6}}}$$

VÝPOČET PRŮSEČÍKŮ:

$\underbrace{y=x^2=2-x}=y$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x_1=-2 \quad x_2=1$

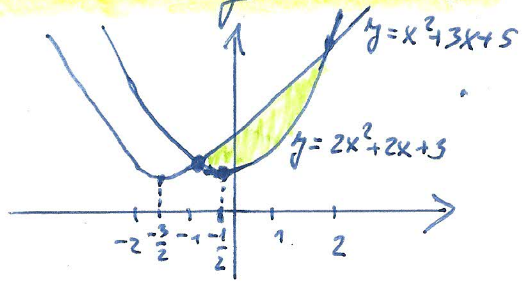
$T_2=[-2,4]$ $T_1=[1,1]$

$y=x^2$ a $x=-3$

$\underbrace{y=9}_{\Downarrow}$

$T_3=[-3,9]$

1. Určete obsah plochy ohraničené křivkami $y=x^2+3x+5$ a $y=2x^2+2x+3$



$$S=\int_{-1}^{2}(x^2+3x+5)-(2x^2+2x+3)\,dx=$$
$$=\int_{-1}^{2}(-x^2+x+2)\,dx=$$
$$=\left[-\frac{x^3}{3}+\frac{x^2}{2}+2x\right]_{-1}^{2}=\left(-\frac{8}{3}+2+4\right)-\left(+\frac{1}{3}+\frac{1}{2}-2\right)=$$
$$=-\frac{9}{3}+6-\frac{1}{2}+2=5-\frac{1}{2}=\underline{\underline{\frac{9}{2}}}$$

$$y=x^2+3x+5=2x^2+2x+3$$
$$0=x^2-x-2=(x-2)(x+1)$$

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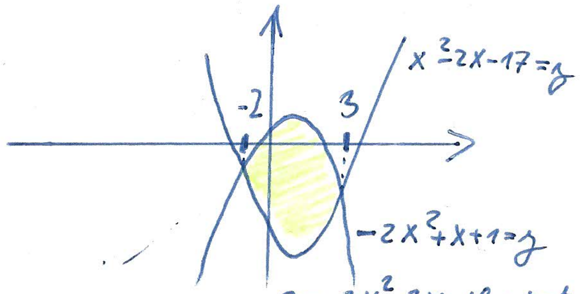
2. $y=x^2-2x-17$, $y=-2x^2+x+1$



$$S=\int_{-2}^{3}(-2x^2+x+1)-(x^2-2x-17)\,dx=$$
$$=\int_{-2}^{3}(-3x^2+3x+18)\,dx=\left[-x^3+\frac{3}{2}x^2+18x\right]_{-2}^{3}=\left(-27+\frac{27}{2}+18\cdot 3\right)-(8+6-2\cdot 18)=\underline{\underline{\frac{125}{2}}}$$

$$0=3x^2-3x-18=y-y$$
$$0=x^2-x-6=(x-3)(x+2)$$

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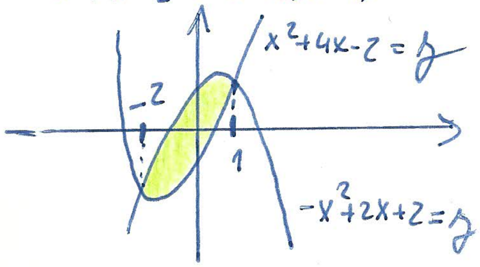
3. $y=x^2+4x-2$ $y=-x^2+2x+2$

$$y=x^2+4x-2=-x^2+2x+2$$
$$2x^2+2x-4=0$$
$$x^2+x-2=(x+2)(x-1)=0$$

$$S=\int_{-2}^{1}(-x^2+2x+2)-(x^2+4x-2)\,dx=$$
$$=\int_{-2}^{1}(-2x^2-2x+4)\,dx=$$
$$=\left[-\frac{2}{3}x^3-x^2+4x\right]_{-2}^{1}=\left(-\frac{2}{3}-1+4\right)-\left(\frac{16}{3}-4-8\right)=\underline{\underline{9}}$$



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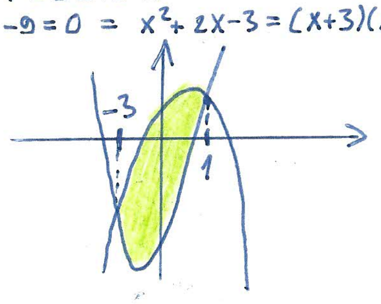
4. $y=2x^2+8x-7$ $y=2x+2-x^2$

$$2x^2+8x-7=2x+2-x^2$$
$$3x^2+6x-9=0=x^2+2x-3=(x+3)(x-1)$$

$$S=\int_{-3}^{1}(2x+2-x^2)-(2x^2+8x-7)\,dx=\int_{-3}^{1}-3x^2-6x+9\,dx=$$
$$=\left[-x^3-3x^2+9x\right]_{-3}^{1}=(-1-3+9)-(\cancel{27}-\cancel{27}-27)=5+27=\underline{\underline{32}}$$



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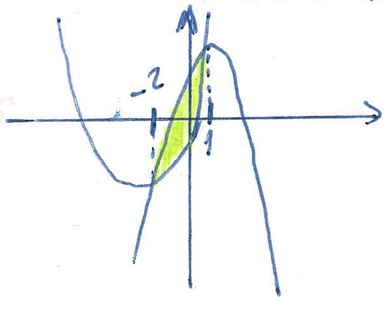
5. $y=x^2+7x-1$ $y=-x^2+5x+3$

$$-x^2+5x+3=x^2+7x-1$$
$$0=2x^2+2x-4$$
$$0=x^2+x-2=(x+2)(x-1)$$

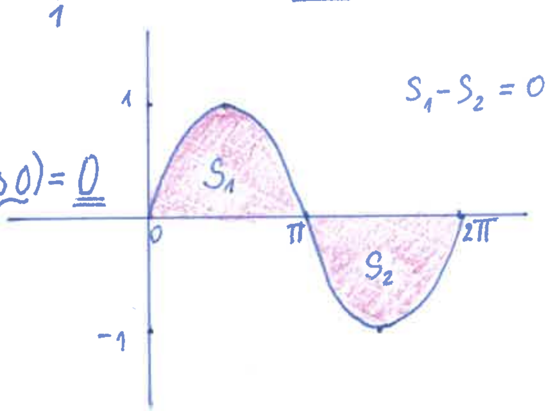
$$S=\int_{-2}^{1}(-x^2+5x+3)-(x^2+7x-1)\,dx=\int_{-2}^{1}-2x^2-2x+4\,dx=$$
$$=\left[-\frac{2}{3}x^3-x^2+4x\right]_{-2}^{1}=\left(-\frac{2}{3}-1+4\right)-\left(\frac{16}{3}-4-8\right)=-6+15=\underline{\underline{9}}$$



Př: Vypočtěte určitý integrál.

$$\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = -\underbrace{\cos\pi}_{-1} - (-\underbrace{\cos 0}_{1}) = -(-1) - (-1) = \underline{\underline{2}}$$

$$\int_0^{2\pi} \sin x \, dx = [-\cos x]_0^{2\pi} = -\underbrace{\cos(2\pi)}_{1} - (-\underbrace{\cos 0}_{1}) = \underline{\underline{0}}$$



$S_1 - S_2 = 0$

Př: Vypočtěte určitý integrál.

$$\int_0^{\frac{\pi}{2}} (3x+1) \sin x \, dx \Rightarrow$$

$$\int (3x+1) \sin x \, dx = \begin{vmatrix} u = 3x+1 & v' = \sin x \\ u' = 3 & v = -\cos x \end{vmatrix} = (3x+1)(-\cos x) - \int 3(-\cos x)\, dx =$$

$$= \underline{(3x+1)(-\cos x) + 3\sin x}$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} (3x+1) \sin x \, dx = \left[(3x+1)(-\cos x) + 3\sin x\right]_0^{\frac{\pi}{2}} =$$

$$= \left(\left(3\tfrac{\pi}{2}+1\right)\left(-\underbrace{\cos\tfrac{\pi}{2}}_{0}\right) + \underbrace{3\sin\tfrac{\pi}{2}}_{3}\right) - \left((3\cdot 0+1)(-\underline{\cos 0}) + 3\sin 0\right) =$$

$$= 3 - (-1) = \underline{\underline{4}}$$

1.) $\int_0^{\frac{\pi}{2}} x\sin(2x)\,dx = I$

$$\int x\cdot\sin(2X)\,dx = \left|\begin{matrix} t=2X \\ dt=2dx\end{matrix}\right| = \int \tfrac12 t\sin t\ \tfrac12 2dx = \tfrac14\int t\sin t\,dt = \left|\begin{matrix}u=t & v'=\sin t\\ u'=1 & v=-\cos t\end{matrix}\right| =$$

$$\tfrac14\left(-t\cos t+\int 1\cos t\,dt\right) = \tfrac14(-t\cos t+\sin t) = \tfrac14(-2x\cos(2x)+\sin(2x))$$

$$I = \tfrac14\left[-2x\cos(2x)+\sin(2x)\right]_0^{\frac{\pi}{2}} = \tfrac14\left(-2\tfrac{\pi}{2}\underbrace{\cos\pi}_{=-1}+\underbrace{\sin\pi}_{=0}\right)-\tfrac14(0+0) = \underline{\underline{\tfrac14\pi}}$$

2.) $\int_1^{e} x^2\ln x\,dx = I$

$$\int x^2\ln x\,dx = \left|\begin{matrix}u'=x^2 & v=\ln x\\ u=\frac{x^3}{3} & v'=\frac1x\end{matrix}\right| = \frac{x^3}{3}\ln x-\int\frac{x^2}{3}dx = \frac{x^3}{3}\ln x-\frac{x^3}{9}$$

$$I = \left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^{e} = \left(\frac{e^3}{3}\underbrace{\ln e}_{=1}-\frac{e^3}{9}\right)-\left(\frac13\underbrace{\ln 1}_{=1}-\frac19\right) = \frac{e^3}{3}-\frac{e^3}{9}+\frac19 = \underline{\underline{\frac29 e^3+\frac19}}$$

3.) $$\int_0^{\pi}(x^2+x+1)\cos x\,dx = \left|\begin{matrix}u=x^2+x+1 & v'=\cos x\\ u'=2x+1 & v=\sin x\end{matrix}\right| = \left[(x^2+x+1)\sin x\right]_0^{\pi}-\int_0^{\pi}(2x+1)\sin x\,dx =$$

$$= \left|\begin{matrix}u=2x+1 & v'=\sin x\\ u'=2 & v=-\cos x\end{matrix}\right| = \left[(x^2+x+1)\sin x\right]_0^{\pi}-\left(\left[(2x+1)(-\cos x)\right]_0^{\pi}+\int_0^{\pi}2\cos x\,dx\right) =$$

$$= \underbrace{\left[(x^2+x+1)\sin x\right]_0^{\pi}}_{=0}+\left[(2x+1)\cos x\right]_0^{\pi}-\underbrace{\left[2\sin x\right]_0^{\pi}}_{=0} =$$

$$= (2\pi+1)\underbrace{\cos\pi}_{=-1}-(2\cdot0+1)\underbrace{\cos 0}_{=1} = -2\pi-1-1 = \underline{\underline{-2\pi-2}}$$

4.) $$\int_0^{\pi}(x^2-3x+2)\sin x\,dx = \left|\begin{matrix}u=x^2-3x+2 & v'=\sin x\\ u'=2x-3 & v=-\cos x\end{matrix}\right| = \left[(x^2-3x+2)(-\cos x)\right]_0^{\pi}+\int_0^{\pi}(2x-3)\cos x\,dx = \left|\begin{matrix}u=2x-3 & v'=\cos x\\ u'=2 & v=\sin x\end{matrix}\right|$$

$$= \left[(x^2-3x+2)(-\cos x)\right]_0^{\pi}+\underbrace{\left[(2x-3)\sin x\right]_0^{\pi}}_{=0}-\left[2(-\cos x)\right]_0^{\pi} = \left[(x^2-3x)(-\cos x)\right]_0^{\pi} =$$

$$= (\pi^2-3\pi)(\underbrace{-\cos\pi}_{=1})-0 = \underline{\underline{\pi^2-3\pi}}$$

5.) $\int_0^{1}(x+1)e^{-x}\,dx = I$

$$\int(x+1)e^{-x}dx = \left|\begin{matrix}t=-x\\ dt=-dx\end{matrix}\right| = \int(x+1)e^{-x}(-1)(-dx) = \int(1-t)e^{t}(-1)dt = \int(t-1)e^{t}dt = \left|\begin{matrix}u=t-1 & v'=e^{t}\\ u'=1 & v=e^{t}\end{matrix}\right| =$$

$$= (t-1)e^{t}-e^{t} = (t-2)e^{t} = (-x-2)e^{-x}$$

$$I = \left[(-x-2)e^{-x}\right]_0^{1} = (-3e^{-1})-(-2e^{0}) = \underline{\underline{-3e^{-1}+2}}$$

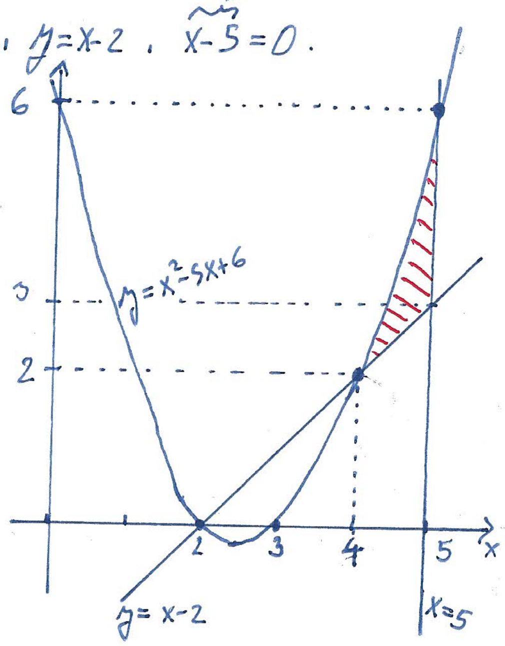
$$\int_0^{1}(x+1)e^{-x}dx = \left|\begin{matrix}u=x+1 & e'=e^{-x}\\ u'=1 & e=-e^{-x}\end{matrix}\right|^{3} = \left[(x+1)e^{-x}\right]_0^{1}-\int_0^{1}(-e^{-x})dx = \left[-(x+1)e^{-x}\right]_0^{1}+\int_0^{1}e^{-x}dx = \left[-(x+1)e^{-x}\right]_0^{1}+\left[-e^{-x}\right]$$

$$= -2e^{-1}-(-e^{0})+(-e^{-1})-(-e^{0}) = \underline{\underline{-3e^{-1}+2}}$$

3. Vypočítejte obsah plochy ohraničené křivkami $y=x^2-5x+6$, $y=x-2$, $x-5=0$. $\Rightarrow x=5$

$$S=\int_4^5 (x^2-5x+6-(x-2))\,dx=\int_4^5 (x^2-6x+8)\,dx=\left[\frac{x^3}{3}-3x^2+8x\right]_4^5=$$

$$=\left(\frac{125}{3}-75+40\right)-\left(\frac{64}{3}-48+32\right)=\frac{61}{3}-19=\underline{\underline{\frac{4}{3}}}$$



4. Vypočítejte obsah plochy ohraničené křivkami $y=2-x^2$ $y-x=0$ $x-3=0$.

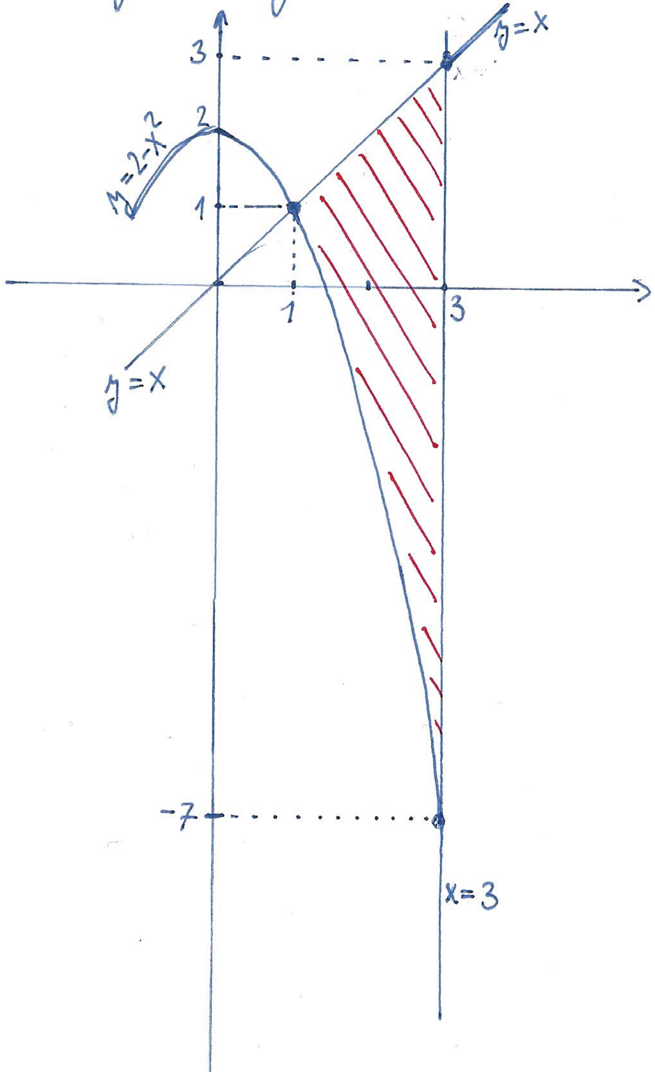
$$S=\int_1^3 (x-(2-x^2))\,dx=$$

$$=\int_1^3 (x^2+x-2)\,dx=$$

$$=\left[\frac{x^3}{3}+\frac{x^2}{2}-2x\right]_1^3=$$

$$=\left(\frac{27}{3}+\frac{9}{2}-6\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)=$$

$$=\frac{26}{3}+\frac{8}{2}-4=\underline{\underline{\frac{26}{3}}}=9-\frac{1}{3}$$



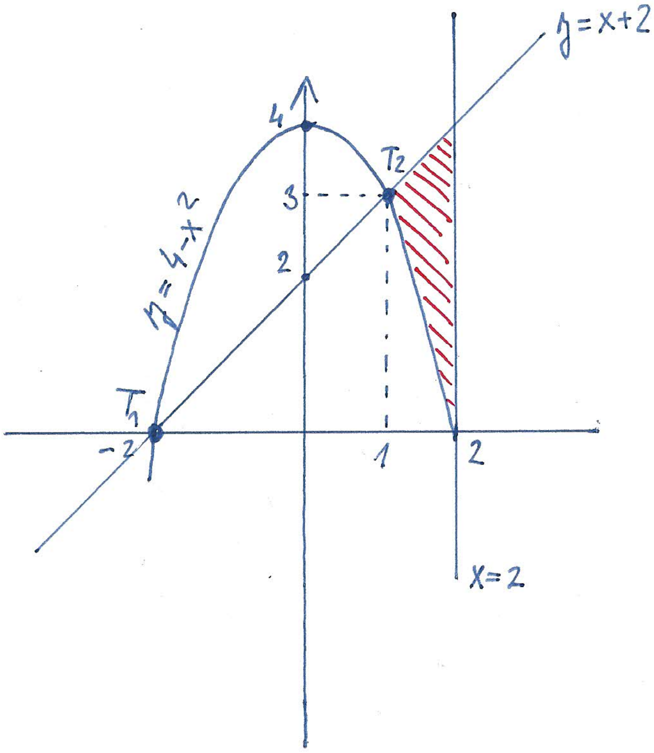
5. Vypočtěte obsah plochy ohraničené křivkami $y = 4 - x^2$, $y = x + 2$, $x - 2 = 0$

$$S = \int_1^2 \left((x+2) - (4 - x^2)\right) dx =$$

$$= \int_1^2 (x^2 + x - 2)\, dx = \left[\frac{x^3}{3} + \frac{x^2}{2} - 2x\right]_1^2 =$$

$$= \left(\frac{8}{3} + \frac{4}{2} - 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= \frac{7}{3} + \frac{3}{2} - 2 = \frac{14 + 9 - 12}{6} = \underline{\underline{\frac{11}{6}}} = 2 - \frac{1}{6}$$



VÝPOČET PRŮSEČÍKŮ:

$$y = \underbrace{4 - x^2 = x + 2}\; = y$$

$$x^2 + x - 2 = 0$$

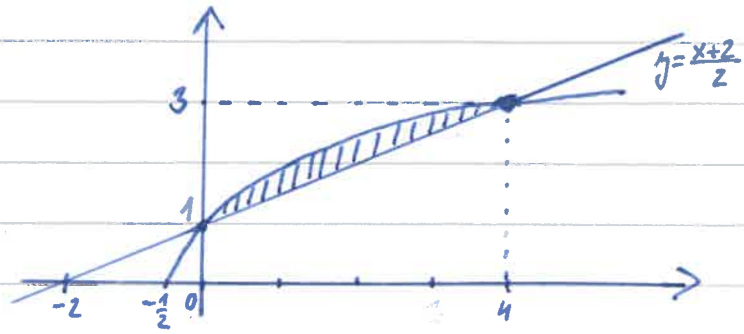
$$(x+2)(x-1) = 0$$

$$x_1 = -2 \quad x_2 = 1$$

$$T_1 = [-2, 0] \quad T_2 = [1, 3]$$

Vypočtěte obsah plochy ohraničené křivkami:

(1.) $y=\sqrt{2x+1}$ a $y=\frac{x+2}{2}$



$$S=\int_0^4\left(\sqrt{2x+1}-\frac{x+2}{2}\right)dx=$$

$$y=\sqrt{2x+1}=\frac{x+2}{2}=y \quad /^2$$
$$2x+1=\frac{x^2+4x+4}{4} \quad /\cdot 4$$
$$8x+4=x^2+4x+4$$
$$0=x^2-4x=(x-4)x \Rightarrow x_{1,2}=\begin{cases}4\\0\end{cases}$$

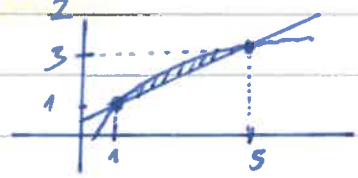
$$I_1=\int\sqrt{2x+1}\,dx=\left|\begin{matrix}t=2x+1\\dt=2dx\end{matrix}\right|=$$
$$=\frac{1}{2}\int\sqrt{2x+1}\,2dx=\frac{1}{2}\int\underbrace{\sqrt{t}}_{t^{\frac{1}{2}}}\,dt=\frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}=$$
$$=\frac{1}{3}(2x+1)^{\frac{3}{2}}$$

$$I_2=\int\frac{x+2}{2}dx=\frac{1}{2}\int(x+2)\,dx=\frac{1}{2}\left(\frac{x^2}{2}+2x\right)=\frac{x^2}{4}+x$$

$$S=\left[\frac{1}{3}(2x+1)^{\frac{3}{2}}-\left(\frac{x^2}{4}+x\right)\right]_0^4=\left(\frac{1}{3}\,9^{\frac{3}{2}}-\left(\frac{16}{4}+4\right)\right)-\left(\frac{1}{3}-0\right)=(9-8)-\frac{1}{3}=\underline{\underline{\frac{2}{3}}}$$

(2.) $y=\sqrt{2x-1}=\sqrt{2(x-1)+1}$; $y=\frac{(x-1)+2}{2}=\frac{x+1}{2}$ (transf. $x'=x-1$)

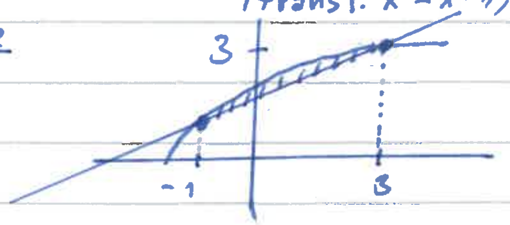
$$S=\int_1^5\left(\sqrt{2x-1}-\frac{x+1}{2}\right)dx=\dots=\underline{\underline{\frac{2}{3}}}$$



(3.) $y=\sqrt{2x+3}=\sqrt{2(x+1)+1}$; $y=\frac{x+3}{2}=\frac{(x+1)+2}{2}$

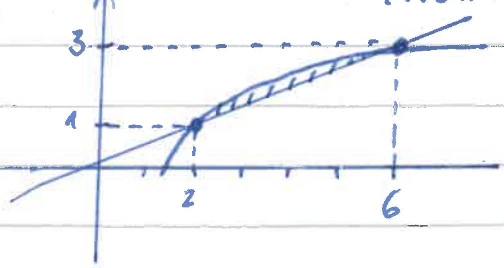
(transf. $x'=x-1$)

$$S=\int_{-1}^3\sqrt{2x+3}-\frac{x+3}{2}\,dx=\dots=\underline{\underline{\frac{2}{3}}}$$

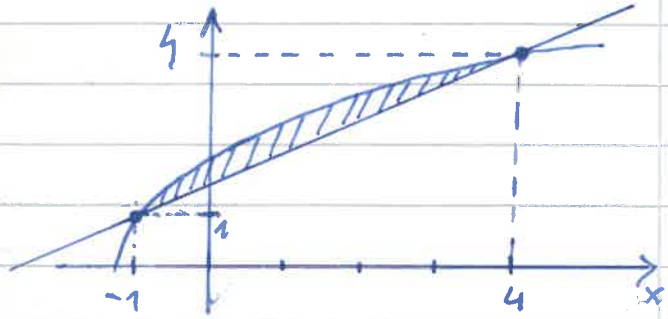


(4.) $y=\sqrt{2x-3}=\sqrt{2(x-2)+1}$; $y=\frac{x}{2}=\frac{(x-2)+2}{2}$ (transf. $x'=x+2$)

$$S=\int_2^6\sqrt{2x-3}-\frac{x}{2}\,dx=\dots=\underline{\underline{\frac{2}{3}}}$$



(5.) $y=\sqrt{3x+4}$ $\qquad y=\frac{3x+8}{5}$



$$S=\int_{-1}^{4}\left(\sqrt{3x+4}-\frac{3x+8}{5}\right)dx$$

$$=\left|\begin{matrix}t=3x+4\\ dt=3dx\end{matrix}\right|=\frac{1}{3}\int_{1}^{16}\Big(\underbrace{\sqrt{t}}_{t^{\frac{1}{2}}}-\frac{t+4}{5}\Big)\,dt=\frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\frac{t^2}{10}-\frac{4t}{5}\right]_1^{16}=$$

$$=\frac{1}{3}\left[\left(\frac{2}{3}\,4^3-\frac{16^2}{10}-\frac{64}{5}\right)-\left(\frac{2}{3}\cdot 1-\frac{1}{10}-\frac{4}{5}\right)\right]=$$

$$=\frac{1}{3}\left[\left(\frac{128}{3}-\frac{128}{5}-\frac{64}{5}\right)-\left(\frac{2}{3}-\frac{1}{10}-\frac{8}{10}\right)\right]=\frac{1}{3}\left[\frac{126}{3}-\frac{192}{5}+\frac{9}{10}\right]=$$

$$=\frac{1}{3}\left[\frac{126}{3}-\frac{384}{10}+\frac{9}{10}\right]=\frac{1}{3}\left[\frac{126}{3}-\frac{375}{10}\right]=\frac{1}{3}\left[\frac{1260-1125}{30}\right]=$$

$$=\frac{1}{3}\left[\frac{135}{30}\right]=\frac{1}{3}\cdot\frac{27}{6}=\frac{27}{3\cdot 3\cdot 2}=\underline{\underline{\frac{3}{2}}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (ax^3+b)\cos(cx)\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} b\cdot\cos(cx)\,dx = \frac{b}{c}\left[\sin(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\int (ax^3+b)\cos(cx)\,dx = \left|\begin{matrix} u = ax^3+b & v' = \cos(cx) \\ u' = 3ax^2 & v = \frac{\sin(cx)}{c}\end{matrix}\right| =$$

$$= \left[(ax^3+b)\frac{\sin(cx)}{c}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3ax^2\sin(cx)\,dx = \left|\begin{matrix} u = 3ax^2 & v' = \sin(cx) \\ u' = 6ax & v = -\frac{\cos(cx)}{c}\end{matrix}\right| =$$

$$= \left[(ax^3+b)\frac{\sin(cx)}{c}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c}\left(\left[3ax^2\left(-\frac{\cos(cx)}{c}\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6ax(-\cos(cx))\,dx\right) =$$

$$= \frac{1}{c}\left[(ax^3+b)\sin(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{c^2}\left[3ax^2\cos(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6ax\cos(cx)\,dx = \left|\begin{matrix} u = 6ax & v' = \cos(cx) \\ u' = 6a & v = \frac{\sin(cx)}{c}\end{matrix}\right|$$

$$= \frac{1}{c}\left[(ax^3+b)\sin(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{c^2}\left[3ax^2\cos(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c^2}\left(\left[6ax\frac{\sin(cx)}{c}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6a\sin(cx)\,dx\right) =$$

$$= \frac{1}{c}\left[(ax^3+b)\sin(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{c^2}\left[3ax^2\cos(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c^3}\left[6ax\sin(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{c^4}\left[6a\cos(cx)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^3-9)\cos(3x)\,dx = \frac{1}{3}\left[(3x^3-9)\sin(3x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{9}\underbrace{\left[9x^2\cos(3x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{=0} - \frac{1}{27}\left[18x\sin(3x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{81}\underbrace{\left[18\cos(3x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{=0}$$

$$= \frac{1}{3}\left[\left(\frac{3\pi^3}{8}-9\right)(-1) - \left(\left(-\frac{3\pi^3}{8}-9\right)\cdot 1\right)\right] - \frac{1}{27}\left[18\frac{\pi}{2}(-1) - \left(18\left(-\frac{\pi}{2}\right)\cdot 1\right)\right] =$$

$$= \frac{1}{3}\left[\cancel{-\frac{3}{8}\pi^3} + 9 + \cancel{\frac{3\pi^3}{8}} + 9\right] - \frac{1}{27}\left[-18\frac{\pi}{2} + 18\frac{\pi}{2}\right] = \underline{\underline{6}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3-2)\cos(3x)\,dx = \underline{\underline{\frac{4}{3}}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-x^3-4)\cos(3x)\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -4\cos(3x) = \left[-\frac{4}{3}\sin(3x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{3} + \frac{4}{3} = \underline{\underline{\frac{8}{3}}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^3+9)\cos(3x)\,dx = -\underline{\underline{6}}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-2x^3+5)\cos(3x)\,dx = -\frac{10}{3}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2x^3-1)\cos(2x) = 0$$

pokud využije lichost $\int x^3\cos(cx)$, ale neokomentuje (nezdůvodní) => 7b