Definiční obor

Př. 1.:

$$f(x) = \sqrt{x^2 - 5x + 6}$$

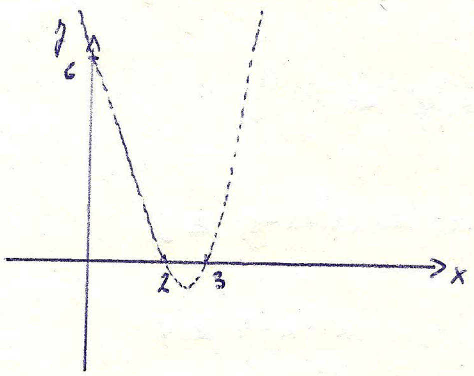
$$x \in D_f \Leftrightarrow x^2 - 5x + 6 \geq 0$$

$$\Updownarrow$$

$$(x-3)(x-2) \geq 0$$

$$\Updownarrow$$

$$x \in D_f \Leftrightarrow \underline{\underline{x \in (-\infty, 2\rangle \cup \langle 3, \infty)}} = D_f$$



Př. 2.:

$$f(x) = \frac{\sqrt{x^2 + 8x + 16}}{x^2 - 3x - 4}$$

$$f(x) = \frac{\sqrt{(x+4)^2}}{(x-4)(x+1)} \Rightarrow x \in D_f \Leftrightarrow \left((x+4)^2 \geq 0\right) \wedge (x \neq 4) \wedge (x \neq -1)$$

$$\Downarrow$$

$$x \in \mathbb{R}$$

$$\underline{\underline{x \in D_f \Leftrightarrow x \in \mathbb{R} - \{-1, 4\}}}$$

Př. 3:

$$f(x) = \frac{\sqrt{e^x \cdot x^2 + 2e^x x + (-3)e^x}}{\sqrt{1 - e^{2x+1}}}$$

$$x \in D_f \Leftrightarrow (e^x \cdot x^2 + 2e^x \cdot x - 3e^x \geq 0) \quad \wedge \quad (1 - e^{2x+1} > 0)$$

$$e^x (x^2 + 2x - 3) \geq 0 \quad /: e^x \qquad\qquad 1 > e^{2x+1}$$

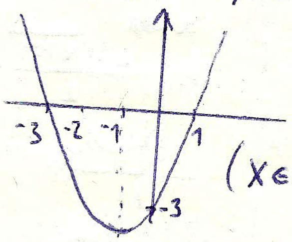
$$x^2 + 2x - 3 \geq 0 \quad \text{(můžeme vydělit, } e^x \neq 0) \qquad\qquad \ln 1 > \ln e^{2x+1}$$

$$(x+3)(x-1) \geq 0 \qquad\qquad 0 > (2x+1)\ln e$$

$$0 > 2x+1$$

$$-1 > 2x$$

$$-\frac{1}{2} > x$$



$$\left(x \in (-\infty, -3\rangle \cup \langle 1, \infty)\right) \quad \wedge \quad \left(x \in (-\infty, -\tfrac{1}{2})\right)$$

$$D_f = \underline{\underline{(-\infty, -3\rangle}}$$

Př. 4: $f(x)=\sqrt{\ln(x^2+x-1)}$

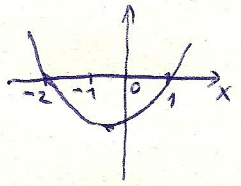
$\Rightarrow \quad x\in D_f \Leftrightarrow$

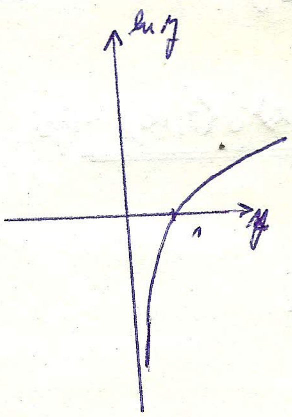
a) $\ln(\underbrace{x^2+x-1}_{y})\ge 0 \quad \wedge$

$x^2+x-1\ge 1$

$x^2+x-2\ge 0$

$(x+2)(x-1)\ge 0$



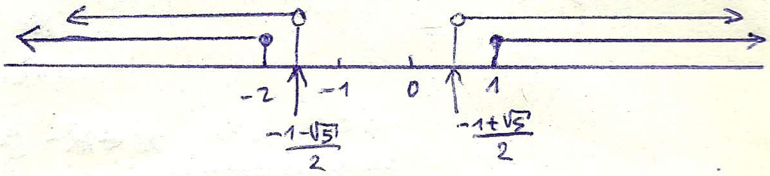


$x\in(-\infty,-2\rangle\cup\langle 1,\infty) \quad \wedge$

b) $x^2+x-1>0$

$x_{1,2}=\dfrac{-1\pm\sqrt{1+4}}{2}=\dfrac{-1\pm\sqrt{5}}{2}\doteq \begin{cases}-1{,}618\\ 0{,}618\end{cases}$

$x\in\left(-\infty,\dfrac{-1-\sqrt5}{2}\right)\cup\left(\dfrac{-1+\sqrt5}{2},\infty\right)$



$\Rightarrow \quad D_f=\underline{\underline{(-\infty,-2\rangle\cup\langle 1,\infty)}}$

Př. 5: $f(x)=\sqrt{\ln(x^2-2|x|+3)}$

$\Rightarrow \quad x\in D_f \Leftrightarrow$

a) $\ln(x^2-2|x|+3)\ge 0 \quad \wedge$

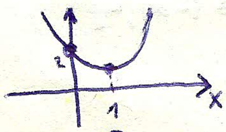
$x^2-2|x|+3\ge 1$

$x^2-2|x|+2\ge 0$

α) $\boxed{x\ge 0}$

$x^2-2x+2\ge 0$

$x_{1,2}=\dfrac{2\pm\sqrt{4-8}}{2} \Rightarrow$ neexistuje



$\Rightarrow \underline{x\in\mathbb{R}}$

nebo

β) $\boxed{x<0}$

$x^2+2x+2\ge 0$

$x_{1,2}=\dfrac{-2\pm\sqrt{4-8}}{2}$ neexistuje

$\Rightarrow \underline{x\in\mathbb{R}}$

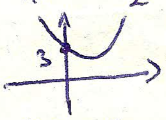
$\Rightarrow x\in\mathbb{R}$

b) $x^2-2|x|+3>0$

α) $x\ge 0$

$x^2-2x+3>0$

$x_{1,2}=\dfrac{2\pm\sqrt{4-12}}{2}$ neexistuje



$\Rightarrow \underline{x\in\mathbb{R}}$

β) $x<0$

$x^2+2x+3>0$

$x_{1,2}=\dfrac{-2\pm\sqrt{4-12}}{2}$ neexistuje

$\Rightarrow \underline{x\in\mathbb{R}}$

$\underline{x\in\mathbb{R}}$

$D_f=\mathbb{R}$

Př 6: $f(x) = \ln|x^2 - x|$

$\Rightarrow x \in D_f \Leftrightarrow \quad |x^2 - x| > 0$

víme, že absolutní hodnota je ≥ 0 => musí být splněno

$x^2 - x \neq 0$

$x(x-1) \neq 0$

$x \neq \begin{cases} 0 \\ 1 \end{cases}$

$\Rightarrow \quad D_f = \mathbb{R} - \{0, 1\}$

Př 7: $f(x) = \arcsin\left(\frac{x-1}{3}\right)$

$x \in D_f \Leftrightarrow \quad \frac{x-1}{3} \in \langle -1, 1 \rangle$

$\Updownarrow$

$-1 \leq \frac{x-1}{3} \leq 1 \qquad / \cdot 3$

$-3 \leq x - 1 \leq 3 \qquad / +1$

$-2 \leq x \leq 4$

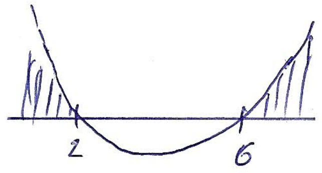
$D_f = \langle -2, 4 \rangle$

Př: Určete def obor funkce $f(x)=\dfrac{\sqrt{x^2-8x+12}}{e^x x-4e^x}$

I) $x^2-8x+12\geq 0$

$(x-6)(x-2)\geq 0$

$x_{1,2}=\begin{cases}6\\2\end{cases}$



$x\in(-\infty,2\rangle\cup\langle 6,\infty)$

II) $e^x x-4e^x\neq 0$

$e^x(x-4)\neq 0 \quad /:e^x>0$

$x-4\neq 0$

$x\neq 4$ tj $x\in\mathbb{R}-\{4\}$

$\Downarrow$

$$x\in(-\infty,2\rangle\cup\langle 6,\infty)=D_f$$

Př: Určete def obor funkce $f(x)=\arccos(x^2-2x+2)$.

$$-1\leq x^2-2x+2\leq 1$$

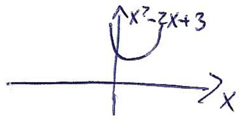
$-1\leq x^2-2x+2$

$0\leq x^2-2x+3$

$x^2-2x+3\geq 0$

$x_{1,2}=\dfrac{2\pm\sqrt{4-12}}{2}$ $\Rightarrow$ kořeny neexistují!

$\Rightarrow$

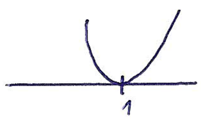


$\Rightarrow$ $x\in\mathbb{R}$

$x^2-2x+2\leq 1$

$x^2-2x+1\leq 0$

$(x-1)^2\leq 0$



$\Rightarrow x\in\{1\}$

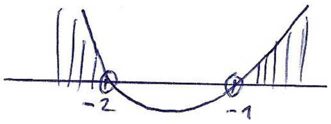
$\Downarrow$

$$x\in\mathbb{R}\cap\{1\}=\{1\}=D_f$$

Př: Určete def. obor funkce $f(x)=\sqrt{-\ln(x^2+3x+2)}$

I.) $x^2+3x+2>0$

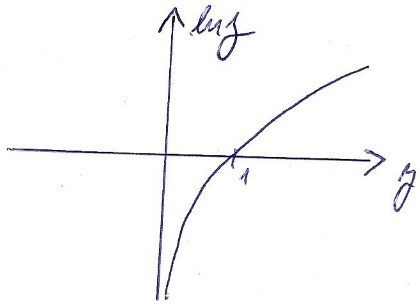
$$x_{1,2}=\frac{-3\pm\sqrt{9-8}}{2}=\begin{cases}\frac{-3+1}{2}=-1\\ \frac{-3-1}{2}=-2\end{cases}$$



$\underline{x\in(-\infty,-2)\cup(-1,\infty)}$

II.) $-\ln(x^2+3x+2)\geq 0$

$$\ln(\overbrace{x^2+3x+2}^{y})\leq 0$$



$$0<x^2+3x+2\leq 1$$

$\underbrace{0<x^2+3x+2,}_{\text{stejná nerovnice jako v I.)}}$

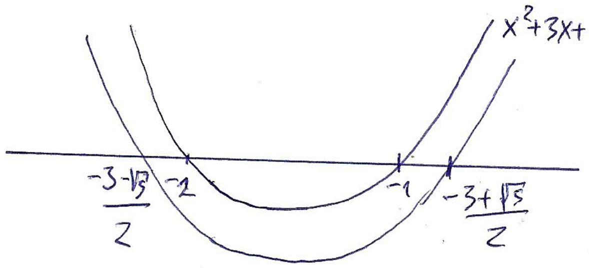
$\underline{x\in(-\infty,-2)\cup(-1,\infty)}$

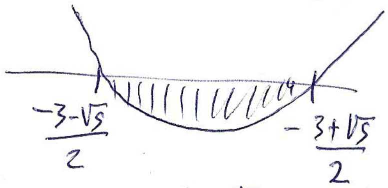
$x^2+3x+2\leq 1$

$x^2+3x+1\leq 0$

$$x_{1,2}=\frac{-3\pm\sqrt{9-4}}{2}=\begin{cases}\frac{-3+\sqrt5}{2}\\ \frac{-3-\sqrt5}{2}\end{cases}$$

$\cancel{x\in\left(-\infty,\frac{-3-\sqrt5}{2}\right\rangle\cup\left\langle\frac{-3+\sqrt5}{2},\infty\right)}$





$\underline{x\in\left\langle\frac{-3-\sqrt5}{2},\frac{-3+\sqrt5}{2}\right\rangle}$

$$\Rightarrow\quad x\in\Big((-\infty,-2)\cup(-1,\infty)\Big)\cap\left\langle\frac{-3-\sqrt5}{2},\frac{-3+\sqrt5}{2}\right\rangle=$$

$$=\underline{\underline{\left\langle\frac{-3-\sqrt5}{2},-2\right)\cup\left(-1,\frac{-3+\sqrt5}{2}\right\rangle}}$$