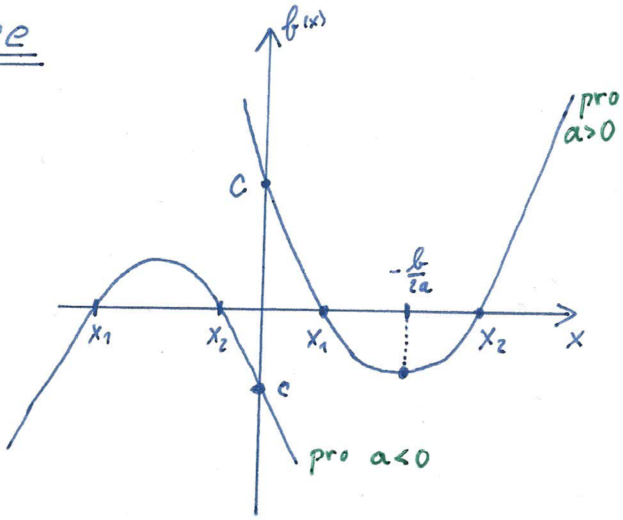
Kvadratická funkce



$f(x) = ax^2 + bx + c \quad ; \; a, b, c \in \mathbb{R}, \; a \neq 0$

grafem je parabola

Kvadratická rovnice: $\quad ax^2 + bx + c = 0$

tuto rovnici splňují: $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ pokud $b^2 - 4ac$ = Diskriminant $= D > 0$

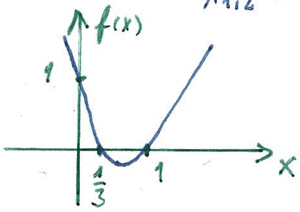
Najdeme tak čísla x_1, x_2, kde $f(x_1) = ax_1^2 + bx_1 + c = 0$ a $f(x_2) = ax_2^2 + bx_2 + c = 0$

$\Rightarrow$ V bodech x_1, x_2 (pokud existují) protíná graf funkce $f(x) = ax^2 + bx + c$ osu X.

Př: Nalezněte všechna $X \in \mathbb{R}$ splňující:

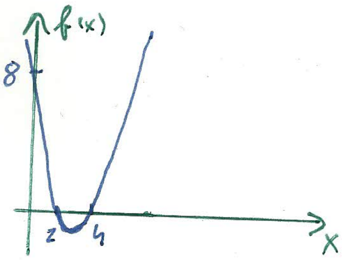
1.) $\underbrace{3X^2 - 4X + 1}_{f(x)} = 0 \quad \Rightarrow \quad X_{1,2} = \dfrac{4 \pm \sqrt{16 - 12}}{6} = \dfrac{4 \pm \sqrt{4}}{6} = \begin{cases} \frac{4+2}{6} = \underline{\underline{1}} \\ \frac{4-2}{6} = \frac{2}{6} = \underline{\underline{\frac{1}{3}}} \end{cases}$

$\Rightarrow$

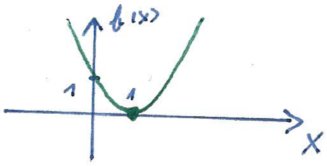


2.) $X^2 - 6X + 8 = 0 \quad \Rightarrow \quad X_{1,2} = \dfrac{6 \pm \sqrt{36 - 32}}{2} = \begin{cases} \underline{\underline{4}} \\ \underline{\underline{2}} \end{cases}$

jinak: $X^2 - 6X + 8 = (X-4) \cdot (X-2) \nearrow$



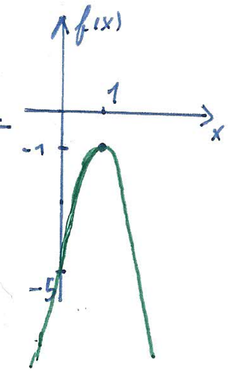
3.) $X^2 - 2X + 1 = 0 \quad \Rightarrow \quad X_{1,2} = \dfrac{2 \pm \sqrt{4-4}}{2} = \dfrac{2 \pm 0}{2} = \underline{\underline{1}}$



4.) $-X^2 + 4X - 5 = 0 \quad \Rightarrow \quad X_{1,2} = \dfrac{-4 \pm \sqrt{16 - 20}}{-2} \Rightarrow X_{1,2}$ $\underline{\underline{\text{neexistuje řešení}}}$

$\shortparallel$

$-(X^2 - 4X + 5) = -\left((X-2)^2 + 1 \right) = -(X-2)^2 - 1$



Př. Načrtněte graf funkce

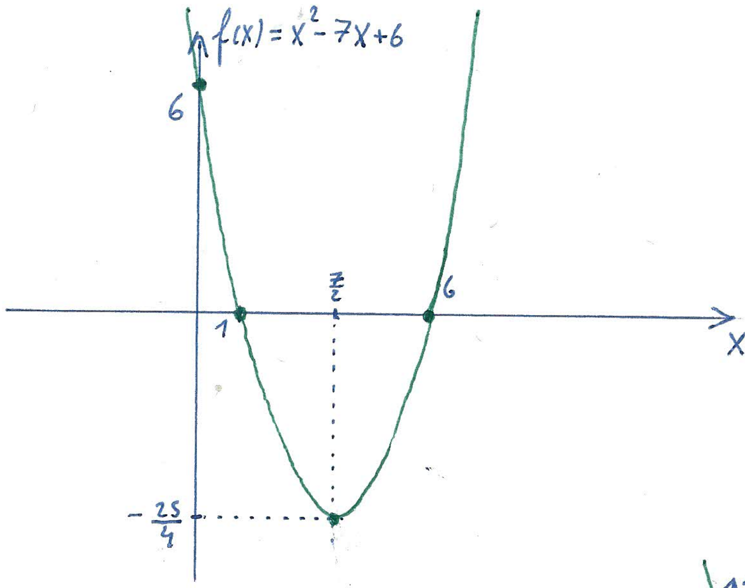
1.) $f(x) = x^2 - 7x + 6$

průsečík s osou y: $f(0) = 0^2 - 7 \cdot 0 + 6 = 6$

průsečíky s osou x: $x_{1,2} = \frac{7 \pm \sqrt{49-24}}{2} = \frac{7 \pm 5}{2} = \left\langle \begin{matrix} 6 \\ 1 \end{matrix} \right.$

souřadnice vrcholu paraboly: $x^2 - 7x + 6 = \underbrace{\left(x - \frac{7}{2}\right)^2}_{x^2 - 7x + \left(\frac{7}{2}\right)^2} - \left(\frac{7}{2}\right)^2 + 6 =$

$= \left(x - \frac{7}{2}\right)^2 + \frac{-49+24}{4} = \left(x - \frac{7}{2}\right)^2 - \frac{25}{4}$



2.) $f(x) = x^2 - 8x + 17$

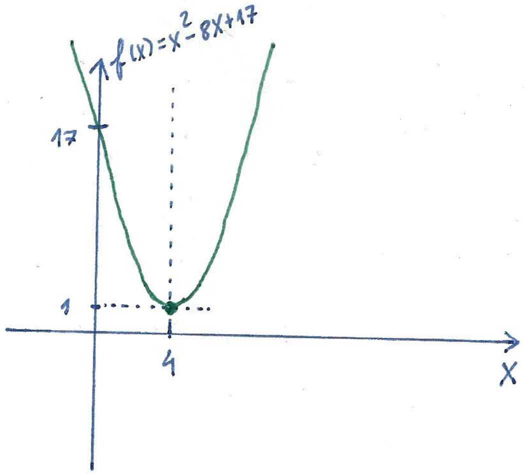
průs. s osou y: $f(0) = 17$

průs. s osou x: $D = 64 - 4 \cdot 17 = -4 \Rightarrow$ neexistují

souřadnice vrcholu: $x^2 - 8x + 17 =$

$= (x-4)^2 - 16 + 17 =$

$= (x-4)^2 + 1$

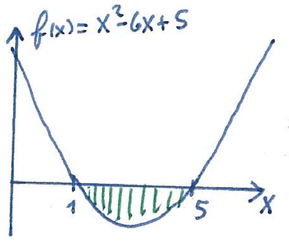


Kvadratické nerovnice: $ax^2+bx+c \gtreqless 0$

Př: Vyřešte kvadratickou nerovnici

1.) $x^2-6x+5<0$

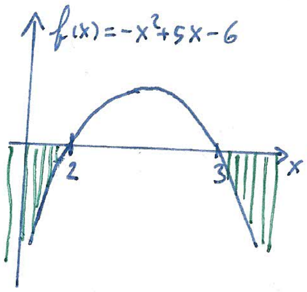
$$x_{1,2}=\frac{6\pm\sqrt{36-20}}{2}=\frac{6\pm4}{2}=\left\langle\begin{matrix}5\\1\end{matrix}\right. \Rightarrow$$



$\Rightarrow$ $\underline{\underline{x\in(1,5)}}$

2.) $-x^2+5x-6\leq 0$

$$x_{1,2}=\frac{-5\pm\sqrt{25-24}}{-2}=\frac{-5\pm1}{-2}=\left\langle\begin{matrix}3\\2\end{matrix}\right. \Rightarrow$$

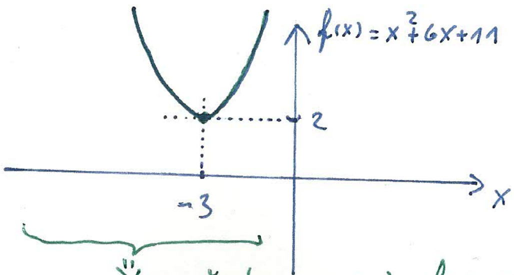


$\Rightarrow$ $\underline{\underline{x\in(-\infty,2\rangle\cup\langle3,\infty)}}$

3.) $x^2+6x+11<0$

$$x_{1,2}=\frac{-6\pm\sqrt{36-4\cdot11}}{2} \Rightarrow \text{průs. neexistuje}$$

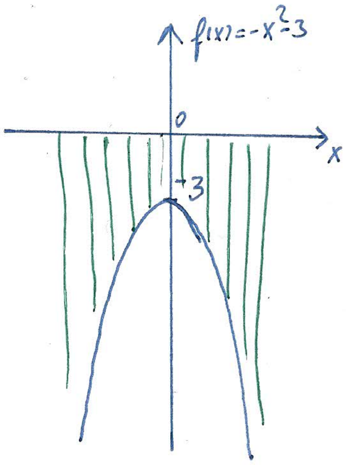
$$x^2+6x+11=(x+3)^2-9+11=(x+3)^2+2$$



$\Downarrow$ pro všechna $x\in\mathbb{R}$ je $f(x)=x^2+6x+11>0$

$\Rightarrow$ řešení neexistuje $\Rightarrow$ $\underline{\underline{x\in\emptyset}}$

4.) $-x^2-3\leq 0$



$\Rightarrow$ $\underline{\underline{x\in\mathbb{R}}}$

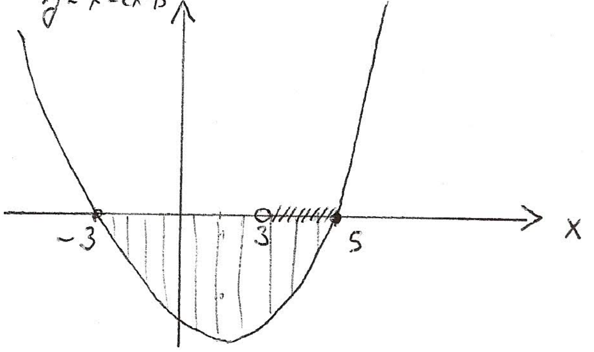
Př: Vyřešte soustavu nerovnic

(1.)
$$x^2 - 2x - 15 \leq 0$$
$$2x - 6 > 0$$

a) $x^2 - 2x - 15 \leq 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4+60}}{2} = \frac{2 \pm \sqrt{64}}{2} = \frac{2 \pm 8}{2} = 1 \pm 4 = \begin{cases} 5 \\ -3 \end{cases}$$

$y = x^2 - 2x - 15$



$\Rightarrow \quad x \in \langle -3, 5 \rangle$

b) $2x - 6 > 0$

$2x > 6$

$x > 3$

$x \in (3, \infty)$

$$x \in \langle -3, 5 \rangle \cap (3, \infty)$$

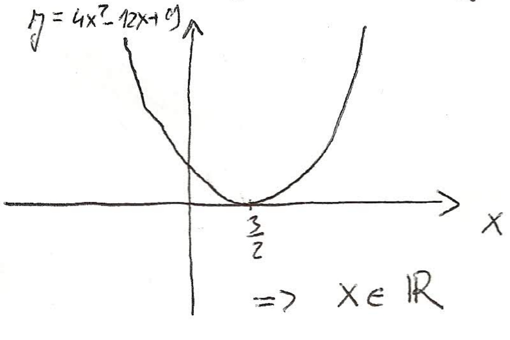
$$\underline{\underline{x \in (3, 5\rangle}}$$

(2.)
$$4x^2 - 12x + 9 \geq 0$$
$$-x^2 + 9 \geq 0$$

a) $4x^2 - 12x + 9 = 0$

$$x_{1,2} = \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{12}{8} = \frac{3}{2}$$

$y = 4x^2 - 12x + 9$



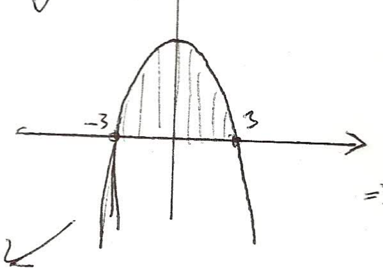
$\Rightarrow \quad x \in \mathbb{R}$

b) $-x^2 + 9 = 0$

$x^2 = 9$

$x = \pm 3$

$y = -x^2 + 9$



$\Rightarrow \quad x \in \langle -3, 3 \rangle$

$$\underline{\underline{x \in \mathbb{R} \cap \langle -3, 3 \rangle = \langle -3, 3 \rangle}}$$

③

$$-(x^2-5x+6) \leq 0$$
$$x^2-9x+20 \leq 0$$

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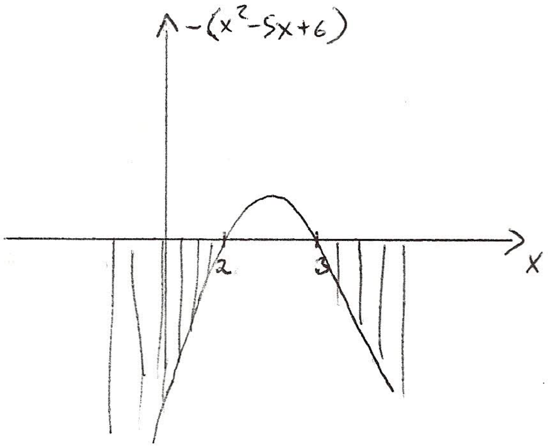
a) $-(x^2-5x+6) \leq 0 \quad / \cdot (-1)$

$x^2-5x+6 \geq 0$

$x^2-5x+6=0$

$x_{1,2} = \dfrac{5 \pm \sqrt{25-24}}{2} = \dfrac{5 \pm 1}{2} = \left\langle \begin{matrix} 3 \\ 2 \end{matrix} \right.$

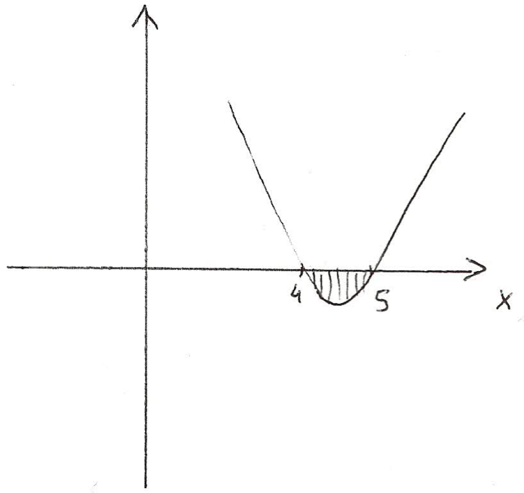


$\Rightarrow \quad x \in (-\infty, 2\rangle \cup \langle 3, \infty)$

b) $x^2-9x+20 \leq 0$

$x^2-9x+20 = 0$

$x_{1,2} = \dfrac{9 \pm \sqrt{81-80}}{2} = \dfrac{9 \pm 1}{2} = \left\langle \begin{matrix} 5 \\ 4 \end{matrix} \right.$



$x \in \langle 4, 5 \rangle$

$$x \in \left[(-\infty, 2) \cup \langle 3, \infty)\right] \cap \langle 4,5\rangle = \underline{\underline{\langle 4,5\rangle}}$$

Př: Nalezněte všechna $x \in \mathbb{R}$ splňující: $x^2 + 3|x| + 2 = 0$.

a) $x \in \langle 0, \infty)$ $\Rightarrow$ $|x| = x$ $\Rightarrow$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0 \Rightarrow x = \begin{cases} -2 \notin \langle 0,\infty) \\ -1 \notin \langle 0,\infty) \end{cases} \Bigg\} \Rightarrow$$ v intervalu $\langle 0, \infty)$ jsme řešení nenašli

b) $x \in (-\infty, 0)$ $\Rightarrow$ $|x| = -x$

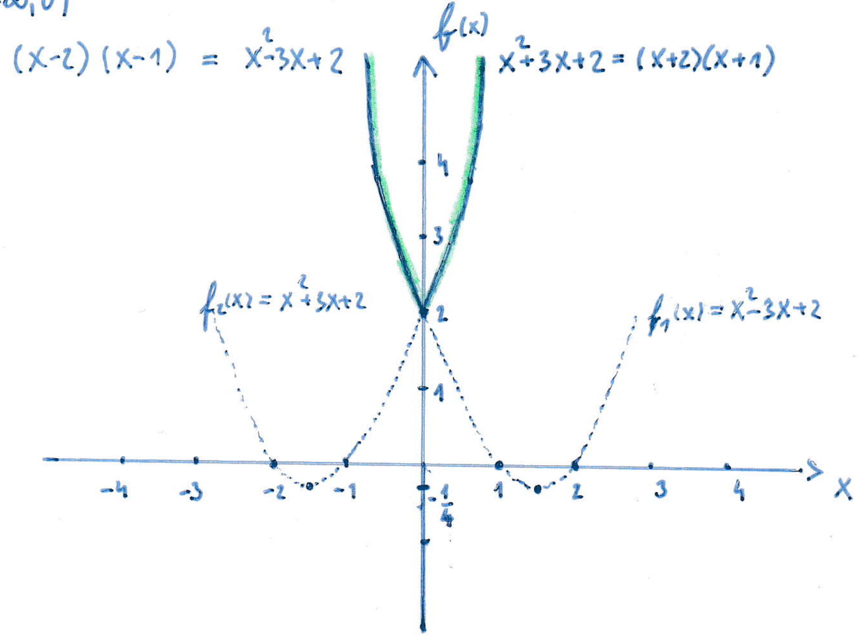
$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0 \Rightarrow x = \begin{cases} 2 \notin (-\infty,0) \\ 1 \in (-\infty,0) \end{cases} \Bigg\} \Rightarrow$$ v intervalu $(-\infty, 0)$ jsme řešení také nenašli

$\Rightarrow$ **Zadaná rovnice nemá řešení (v $\mathbb{R}$).**

Př: načrtněte graf funkce $f(x) = x^2 + 3|x| + 2$

$$f(x) = \begin{cases} x^2 + 3x + 2 & \text{pro } x \in \langle 0, \infty) \\ x^2 - 3x + 2 & \text{pro } x \in (-\infty, 0) \end{cases}$$



$f(0) = 2$

$f_1(\frac{3}{2}) = (\frac{3}{2})^2 - 3\frac{3}{2} + 2 = \frac{9-18+8}{4} = -\frac{1}{4}$

$f_2(-\frac{3}{2}) = (-\frac{3}{2})^2 + 3(-\frac{3}{2}) + 2 = \frac{9-18+8}{4} = -\frac{1}{4}$

(je to sudá funkce)

Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $-x^2 + 2|x+2| - 4 = 0$

a) $x \in \langle -2, \infty)$ $\Rightarrow$ $|x+2| = x+2$

$$-x^2 + 2(x+2) - 4 = 0$$
$$-x^2 + 2x + 4 - 4 = 0$$
$$-x^2 + 2x = 0$$
$$-x(x-2) = 0 \Rightarrow x = \begin{cases} \underline{\underline{0}} \in \langle -2, \infty) \\ \underline{\underline{2}} \in \langle -2, \infty) \end{cases}$$

b) $x \in (-\infty, -2)$ $\Rightarrow$ $|x+2| = -(x+2)$

$$-x^2 - 2(x+2) - 4 = 0$$
$$-x^2 - 2x - 4 - 4 = 0$$
$$-x^2 - 2x - 8 = 0 \quad / \cdot (-1)$$
$$x^2 + 2x + 8 = 0$$
$$x_{1,2} = \frac{-2 \pm \sqrt{\underbrace{4 - 4\cdot 8}_{=D}}}{2} \Rightarrow D < 0 \Rightarrow \text{žádné řešení v } \mathbb{R}!$$

$\Rightarrow$ **Všechna řešení rovnice jsou $x=0$ a $x=2$.**

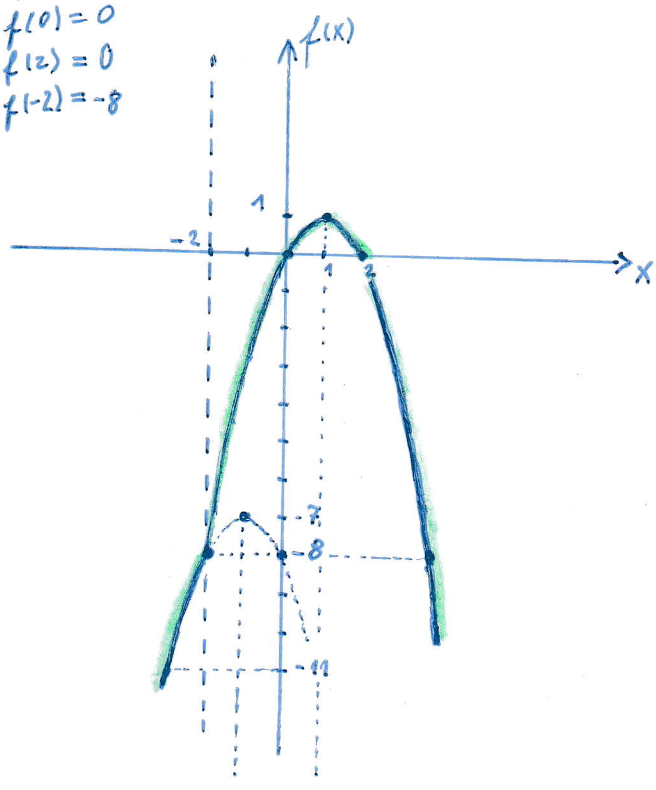
Př. Načrtněte graf funkce $f(x) = -x^2 + 2|x+2| - 4$

a) pro $x \in \langle -2, \infty)$ je $f(x) = -x^2 + 2(x+2) - 4 =$
$= -x^2 + 2x = -x(x-2)$

$\Big\}$ $f(1) = 1$
$f(0) = 0$
$f(2) = 0$
$f(-2) = -8$

b) pro $x \in (-\infty, -2)$ je $f(x) = -x^2 - 2(x+2) - 4 =$
$= -x^2 - 2x - 8 =$
$= -(x^2 + 2x + 8) =$
$= -((x+1)^2 + 7)$

$\Rightarrow f(-3) = -((-3+1)^2 + 7) = -11$



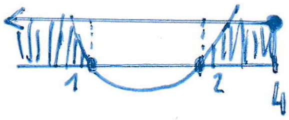
Př. *Nalezněte všechna $x \in \mathbb{R}$ splňující* $x^2 + 3|x-4| - 10 \geq 0$

$x \in (-\infty, 4\rangle$	$x \in \langle 4, \infty)$
$\lvert x-4\rvert = -(x-4)$	$\lvert x-4\rvert = (x-4)$

$x^2 - 3(x-4) - 10 \geq 0$

$x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$

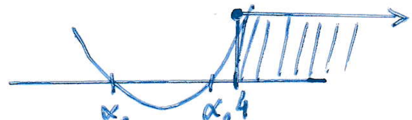


$x \in (-\infty, 1\rangle \cup \langle 2, 4\rangle$

$x^2 + 3(x-4) - 10 \geq 0$

$x^2 + 3x - 22 \geq 0$

$$x_{1,2} = \frac{-3 \pm \sqrt{9+88}}{2} = \begin{cases} \frac{-3+\sqrt{97}}{2} \doteq 3{,}42 = \alpha_1 \\ \frac{-3-\sqrt{97}}{2} \doteq -6{,}42 = \alpha_2 \end{cases}$$

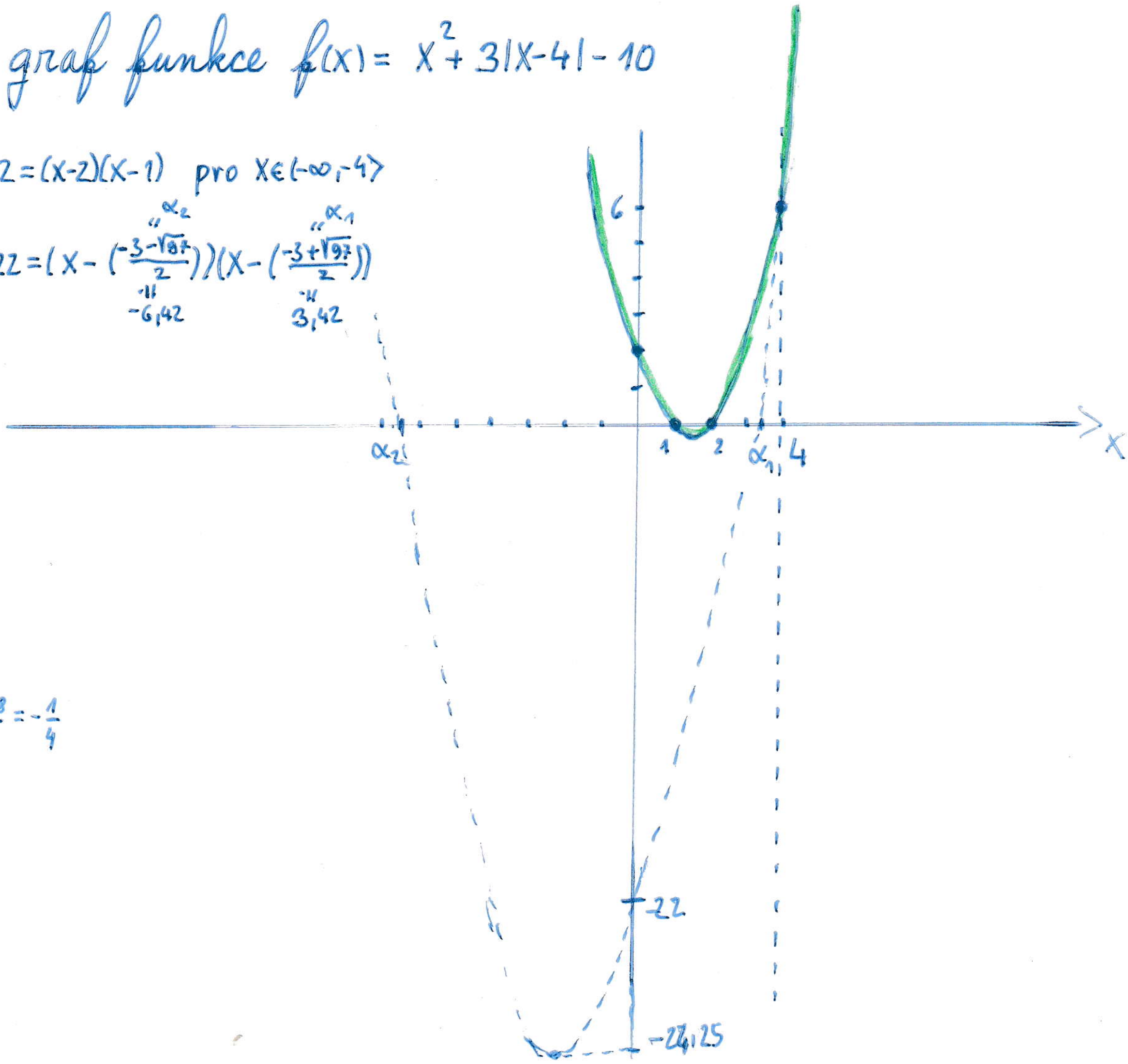


$x \in \langle 4, \infty)$

$\Rightarrow$ $\underline{\underline{x \in (-\infty, 1\rangle \cup \langle 2, \infty)}}$

Př. *Načrtněte graf funkce* $f(x) = x^2 + 3|x-4| - 10$

$$f(x) = \begin{cases} x^2 - 3x + 2 = (x-2)(x-1) \quad \text{pro } x \in (-\infty, 4\rangle \\ x^2 + 3x - 22 = \Big(x - \underbrace{\big(\tfrac{-3-\sqrt{97}}{2}\big)}_{\doteq\, -6{,}42}^{\text{„}\alpha_2\text{"}}\Big)\Big(x - \underbrace{\big(\tfrac{-3+\sqrt{97}}{2}\big)}_{\doteq\, 3{,}42}^{\text{„}\alpha_1\text{"}}\Big) \end{cases}$$



$x^2 + 3x - 22 = -24{,}25$

$f\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9-18+8}{4} = -\frac{1}{4}$

Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $-x^2 + 3|x-4| - 8 \geq 0$

$x \in (-\infty, 4\rangle$	$x \in (4, \infty)$
$\lvert x-4 \rvert = -(x-4)$	$\lvert x-4 \rvert = x-4$

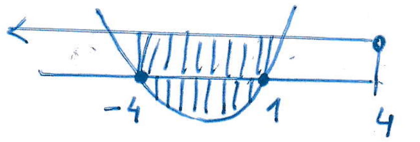
$x \in (-\infty, 4\rangle$:

$$-x^2 - 3(x-4) - 8 \geq 0$$

$$-x^2 - 3x + 4 \geq 0 \quad /\cdot(-1)$$

$$x^2 + 3x - 4 \leq 0$$

$$(x+4)(x-1) \leq 0$$



$x \in \langle -4, 1 \rangle$

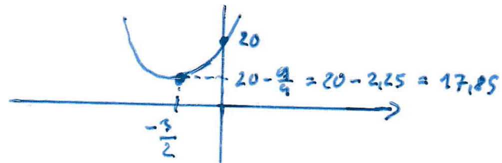
$x \in (4, \infty)$:

$$-x^2 + 3(x-4) - 8 \geq 0$$

$$-x^2 + 3x - 20 \geq 0$$

$$x^2 - 3x + 20 \leq 0$$

$$\left(x - \tfrac{3}{2}\right)^2 - \tfrac{9}{4} + 20 \leq 0$$



$x \in \emptyset$

$$\Rightarrow \quad \underline{\underline{x \in \langle -4, 1 \rangle}}$$

Př. Načrtněte graf funkce $f(x) = -x^2 + 3|x-4| - 8$

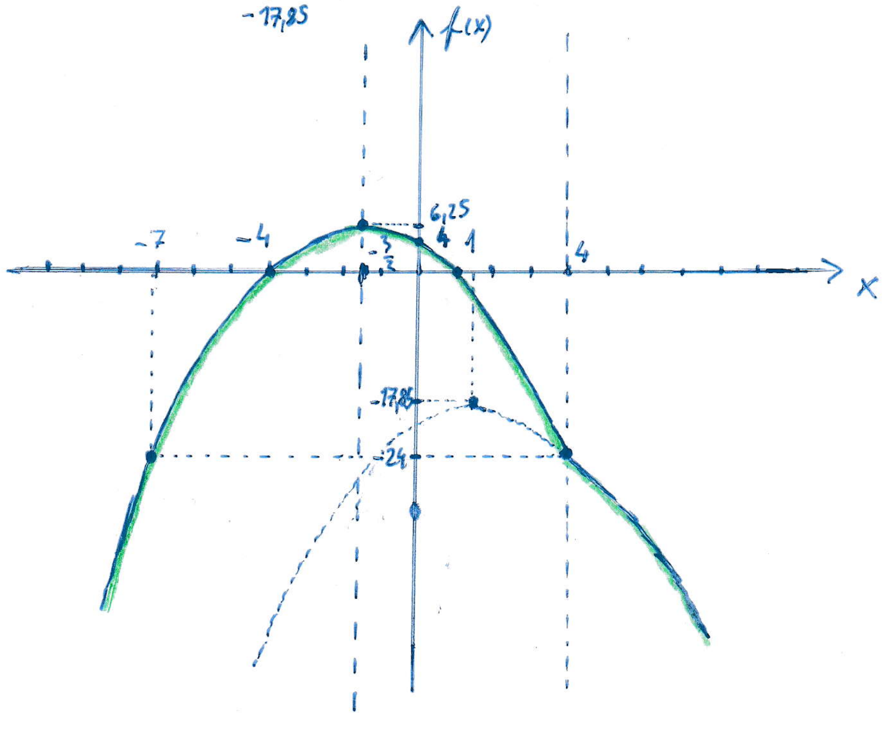
$$f(x) = \begin{cases} -x^2 - 3(x-4) - 8 = -x^2 - 3x + 4 = -(x+4)(x-1) & \text{pro } x \in (-\infty, 4\rangle \\ -x^2 + 3(x-4) - 8 = -x^2 + 3x - 20 = (x - \tfrac{3}{2})^2 + \underbrace{\tfrac{9}{4} - 20}_{-17,85} & \text{pro } x \in (4, \infty) \end{cases}$$

$f(4) = -16 - 8 = -24$

$f(0) = -8$

$f(-\tfrac{3}{2}) = -\tfrac{9}{4} + 3\,\tfrac{3}{2} + 4 =$

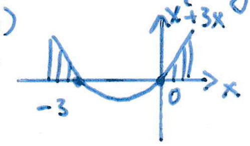
$= \dfrac{-9 + 18 + 16}{4} = \dfrac{25}{4} = 6{,}25$



Př. Nalezněte všechna řešení ($x \in \mathbb{R}$) rovnice $|x^2+3x|+x-5=0$

a) $x^2+3x \geq 0$ je splněno pro $x \in (-\infty,-3\rangle \cup \langle 0,\infty)$

$\underset{x\cdot(x+3)}{\parallel}$



$\Rightarrow$ Pro tato x je $|x^2+3x| = (x^2+3x)$ $\Rightarrow$ Řešíme rovnici:

$x^2+3x+x-5=0$

$x^2+4x-5=0$

$(x+5)(x-1)=0 \Rightarrow x = \begin{cases} \underline{\underline{-5}} \in (-\infty,-3\rangle\cup\langle 0,\infty) \Rightarrow \text{je to řešení} \\ \underline{\underline{1}} \in (-\infty,-3\rangle\cup\langle 0,\infty) \Rightarrow \text{je to řešení} \end{cases}$

b) $x^2+3x<0$ je splněno pro $x \in (-3,0)$ $\Rightarrow$ pro tato x řešíme:

$-(x^2+3x)+x-5=0$

$-x^2-2x-5=0$

$-(x^2+2x+5)=0$

$-((x+1)^2+4)=0 \Rightarrow$ v $\mathbb{R}$ nemá řešení

$\Rightarrow$ Zadaná rovnice je splněna jen pro $x=-5$ a $x=1$.

Př. Načrtněte graf funkce $f(x) = |x^2+3x|+x-5$

$f(x) = \begin{cases} x^2+4x-5 = (x+5)(x-1) & \text{pro } x \in (-\infty,-3\rangle\cup\langle 0,\infty) \\ -((x+1)^2+4) & \text{pro } x \in (-3,0) \text{ (ale i pro } x=3 \text{ a } x=0) \end{cases}$

$f(1)=0$

$f(-5)=0$

$f(0)=|0+0|+0-5=-5$

$f_1(x)=x^2+4x-5$ $\Rightarrow f_1(-2)=4-8-5=-9$

$\underset{(x+5)(x-1)}{\parallel}$

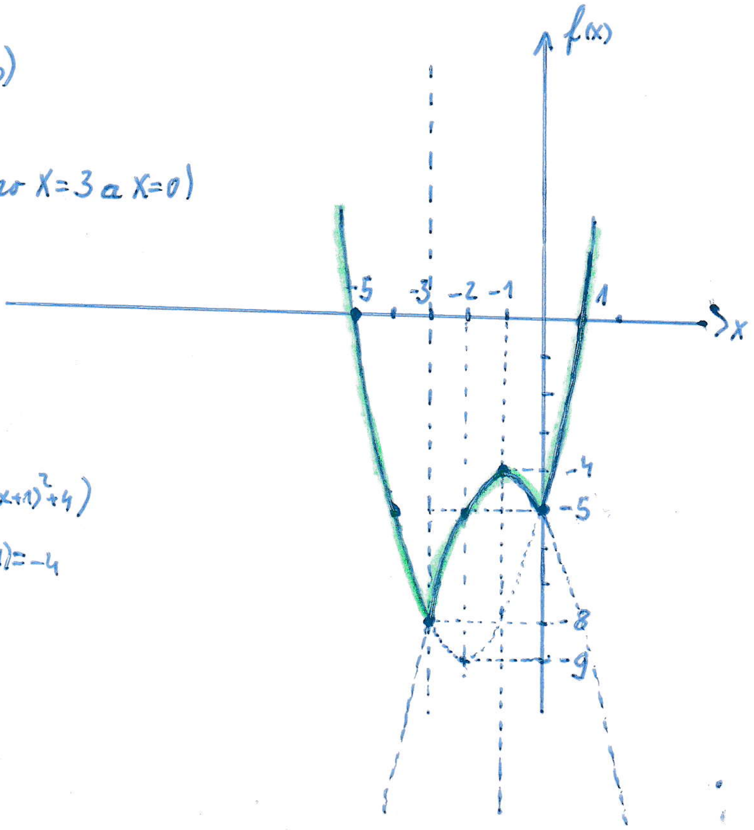
$f_1(0)=-5$

$f_1(-3)=9-12-5=-8$

$f_1(-4)=-5$

$f_2(x)=-((x+1)^2+4)$

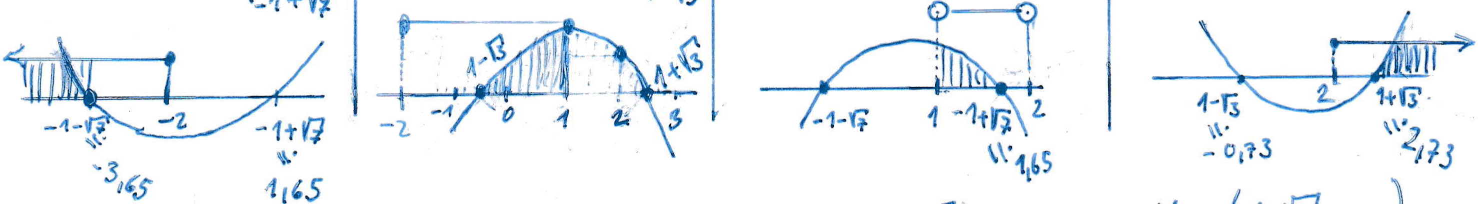
$f_2(-1)=-4$



Př: Nalezněte všechna $x \in \mathbb{R}$ splňující $|x^2-4| - 2|x-1| \ge 0$.

$$|x^2-4| \begin{cases} (x^2-4) & \text{pro } x \in (-\infty,-2\rangle \cup \langle 2,\infty) \\ -(x^2-4) & \text{pro } x \in (-2,2) \end{cases} \qquad |x-1| \begin{cases} (x-1) & \text{pro } x \in (1,\infty) \\ -(x-1) & \text{pro } x \in (-\infty,1\rangle \end{cases}$$

$x \in (-\infty,-2\rangle$	$x \in (-2,1\rangle$	$x \in (1,2)$	$x \in \langle 2,\infty)$
$\lvert x^2-4\rvert = x^2-4$ $\lvert x-1\rvert = -(x-1)$	$\lvert x^2-4\rvert = -(x^2-4)$ $\lvert x-1\rvert = -(x-1)$	$\lvert x^2-4\rvert = -(x^2-4)$ $\lvert x-1\rvert = x-1$	$\lvert x^2-4\rvert = x^2-4$ $\lvert x-1\rvert = x-1$
$(x^2-4)+2(x-1) \ge 0$ $x^2+2x-6 \ge 0$ $x_{1,2} = \frac{-2\pm\sqrt{4+24}}{2} = -1-\sqrt{7},\ -1+\sqrt{7}$	$-(x^2-4)+2(x-1) \ge 0$ $-x^2+2x+2 \ge 0$ $x_{1,2} = \frac{-2\pm\sqrt{4+8}}{-2} = 1-\sqrt{3},\ 1+\sqrt{3}$	$-(x^2-4)-2(x-1)$ $-x^2-2x+6 \ge 0$ $x_{1,2} = \frac{2\pm\sqrt{4+24}}{-2} = -1-\sqrt{7},\ -1+\sqrt{7}$	$(x^2-4)-2(x-1) \ge 0$ $x^2-2x-2 \ge 0$ $x_{1,2} = \frac{2\pm\sqrt{4+8}}{2} = 1-\sqrt{3},\ 1+\sqrt{3}$



$x \in (-\infty,-1-\sqrt{7}\rangle$	$x \in \langle 1-\sqrt{3},1\rangle$	$x \in (1,-1+\sqrt{7}\rangle$	$x \in \langle 1+\sqrt{3},\infty)$

$$\Rightarrow \underline{\underline{x \in (-\infty,-1-\sqrt{7}\rangle \cup \langle 1-\sqrt{3},-1+\sqrt{7}\rangle \cup \langle 1+\sqrt{3},\infty)}}$$

Př: Načrtněte graf funkce $f(x) = |x^2-4| - 2|x-1|$

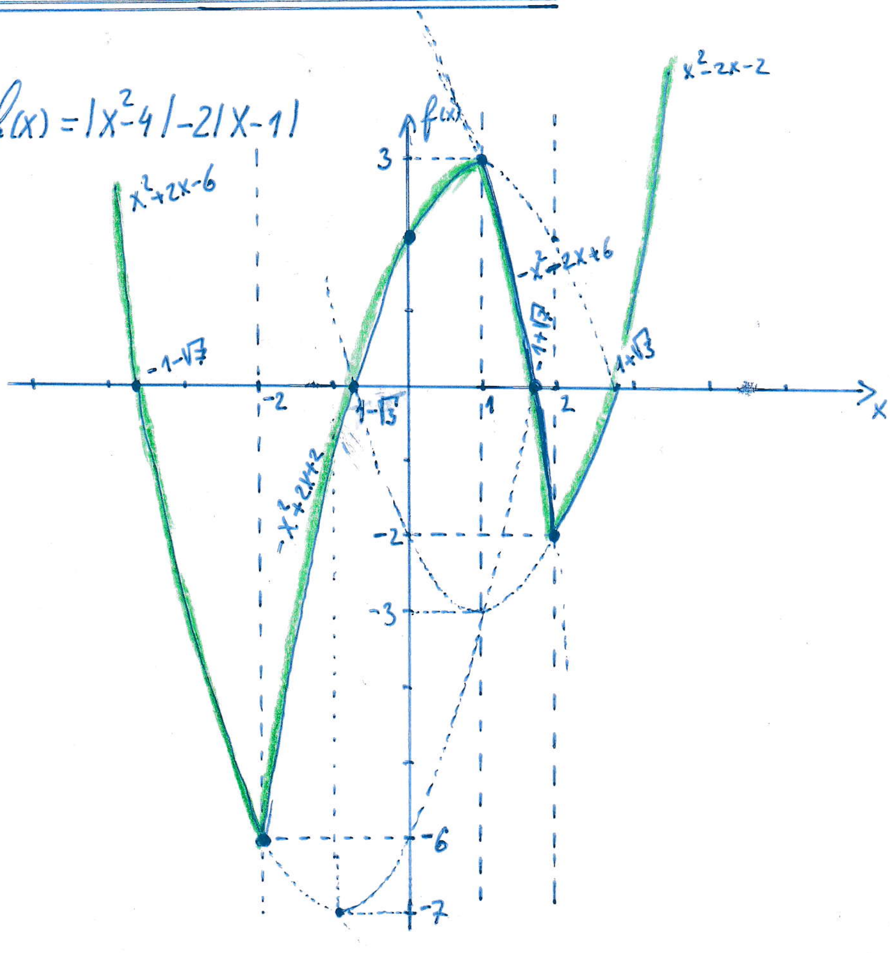
$$f(x) = \begin{cases} x^2+2x-6 & \text{pro } x \in (-\infty,-2\rangle \\ -x^2+2x+2 & \text{pro } x \in (-2,1\rangle \\ -x^2-2x+6 & \text{pro } x \in (1,2) \\ x^2-2x-2 & \text{pro } x \in \langle 2,\infty) \end{cases}$$

$f(-2) = -6$

$f(1) = 3$

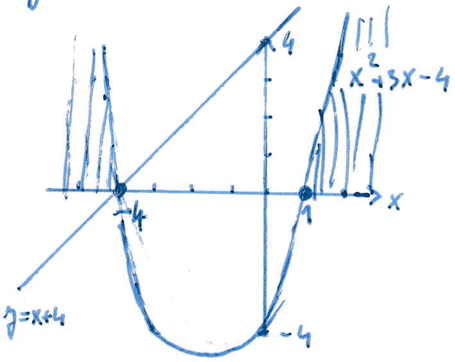
$f(2) = -2$

$f(0) = 2$



Př. Naleznete všechna $x \in \mathbb{R}$ splňující: $|x^2+3x-4|-|x+4| \geq 0$.

$x^2+3x-4 = (x+4)(x-1) \quad \Rightarrow$

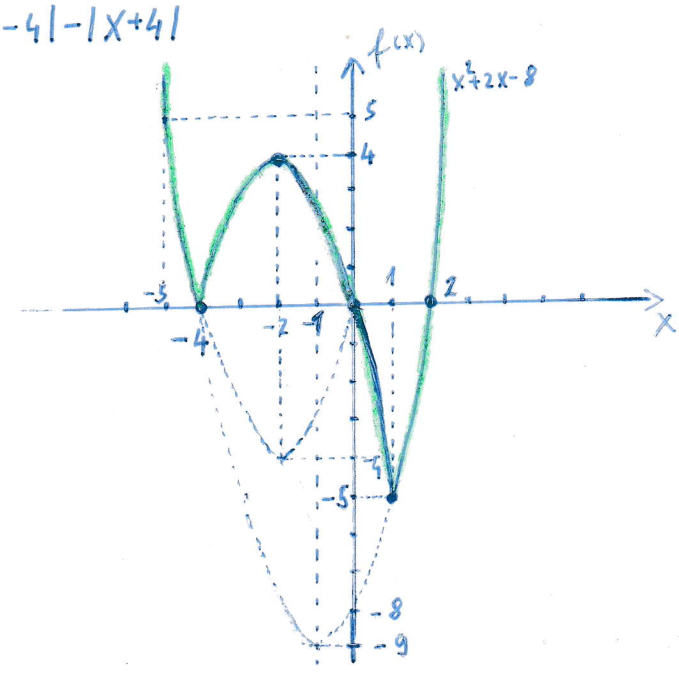


$x \in (-\infty,-4\rangle$	$x \in (-4,1\rangle$	$x \in (1,\infty)$
$\lvert x^2+3x-4\rvert = x^2+3x-4$	$\lvert x^2+3x-4\rvert = -x^2-3x+4$	$\lvert x^2+3x-4\rvert = x^2+3x-4$
$\lvert x+4\rvert = -x-4$	$\lvert x+4\rvert = x+4$	$\lvert x+4\rvert = x+4$
$x^2+3x-4-(-x-4) \geq 0$	$-x^2-3x+4-(x+4) \geq 0$	$x^2+3x-4-(x+4) \geq 0$
$x^2+4x \geq 0$	$-x^2-4x \geq 0 \quad \lvert \cdot(-1)$	$x^2+2x-8 \geq 0$
$x(x+4) \geq 0$	$x^2+4x \leq 0$	$(x+4)(x-2) \geq 0$
	$x(x+4) \leq 0$	
$x \in [(-\infty,-4\rangle \cup \langle 0,\infty)] \cap (-\infty,-4\rangle$	$x \in \langle -4,0\rangle \cap (-4,1\rangle$	$x \in [(-\infty,-4\rangle \cup \langle 2,\infty)] \cap (1,\infty)$
$x \in (-\infty,-4\rangle$	$x \in \langle -4,0\rangle$	$x \in \langle 2,\infty)$

$\Rightarrow$ Nerovnice je splněna pro $x \in \overbrace{(-\infty,-4\rangle \cup \langle -4,0\rangle}^{(-\infty,0\rangle} \cup \langle 2,\infty)$

Př. Načrtněte graf funkce $f(x) = |x^2+3x-4|-|x+4|$

$$f(x) = \begin{cases} x(x+4) & \text{pro } x \in (-\infty,-4\rangle \\ -x^2-4x = -x(x+4) & \text{pro } x \in (-4,1\rangle \\ x^2+2x-8 = (x+4)(x-2) & \text{pro } x \in (1,\infty) \end{cases}$$



Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $|1+x|\cdot|1-x| = x\cdot|x|$

$1-x \geq 0 \iff 1 > x$

$x\in(-\infty,-1\rangle$ $\quad -1$	$x\in(-1,0\rangle$ $\quad 0$	$x\in(0,1\rangle$ $\quad 1$	$x\in(1,\infty)$
$\lvert 1+x\rvert=-(1+x)$ $\lvert 1-x\rvert=1-x$ $\lvert x\rvert=-x$	$\lvert 1+x\rvert=1+x$ $\lvert 1-x\rvert=1-x$ $\lvert x\rvert=-x$	$\lvert 1+x\rvert=1+x$ $\lvert 1-x\rvert=1-x$ $\lvert x\rvert=x$	$\lvert 1+x\rvert=1+x$ $\lvert 1-x\rvert=-(1-x)$ $\lvert x\rvert=x$
$-(1+x)(1-x)=-x^2$ $-(1-x^2)=-x^2$ $-1+x^2=-x^2$ $1=2x^2$ $x=\pm\frac{1}{\sqrt2}\doteq\pm0{,}707$ $\notin(-\infty,-1\rangle$ $\emptyset$	$(1+x)(1-x)=-x^2$ $1-x^2=-x^2$ $1=0$ $\emptyset$	$(1+x)(1-x)=x^2$ $1-x^2=x^2$ $1=2x^2$ $x^2=\frac12$ $x = -\frac{1}{\sqrt2}\notin(0,1\rangle$ $x = \frac{1}{\sqrt2}\in(0,1\rangle$	$-(1+x)(1-x)=x^2$ $-(1-x^2)=x^2$ $-1+x^2=x^2$ $-1=0$ $\emptyset$

$$\Rightarrow \quad \underline{\underline{x=\frac{1}{\sqrt2}}}$$

Př. Do jednoho obrázku namalujte grafy funkcí $f(x)=|1+x||1-x|$ a $g(x)=x\cdot|x|$ a $f(x)-g(x)=h(x)$ na další obrázek

$$f(x)=|1-x^2|=\begin{cases}1-x^2 & \text{pro } x\in\langle-1,1\rangle\\ -1+x^2 & \text{pro } x\in(-\infty,-1)\cup(1,\infty)\end{cases}$$

$$g(x)=\begin{cases}x^2 & \text{pro } x\geq0\\ -x^2 & \text{pro } x<0\end{cases}$$

$$h(x)=\begin{cases}-1+2x^2 & \text{pro } x\in(-\infty,-1\rangle\\ 1 & \text{pro } x\in(-1,0\rangle\\ 1-2x^2 & \text{pro } x\in(0,1\rangle\\ -1 & \text{pro } x\in(1,\infty)\end{cases}$$