A

1.) Je dána matice $A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -3 & 2 \\ 1 & 5 & 0 \end{pmatrix}$. Určete matici k ní inverzní.

2.) Vyřešte soustavu lineárních rovnic:

$$\begin{aligned} x_1 - x_2 + 2x_3 + x_4 &= 3 \\ 3x_1 - 2x_2 + x_3 + x_4 &= 3 \\ -x_1 + 3x_3 + x_4 &= 3 \\ x_1 - 2x_2 + 7x_3 + 3x_4 &= 9 \end{aligned}$$

3.) Lineární zobrazení $A : \mathbb{R}^3 \to \mathbb{R}^2$ je dáno svými hodnotami $A((1,1,0)) = (1,-1)$, $A((1,2,1)) = (0,1)$, $A((0,-1,5)) = (3,2)$. Určete $A((-1,-4,3))$.

4.) Určete souřadnice polynomu (vektoru) $-3x^2 + 5x$ vzhledem k bázi $E = \{-x^2 + 2x + 1,\ -x^2 - x, x - 1\}$

5.) Vypočtěte součin matic:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & 5 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

B

1.) Vynásobte $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 2 & 3 & 1 \end{pmatrix}$

2.) Je dána matice $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$. Určete matici k ní inverzní.

3.) Vyřešte soustavu lineárních rovnic:

$$\begin{aligned} x_1 + x_2 - 2x_3 + 3x_4 &= 3 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 10 \\ x_1 + x_3 + 2x_4 &= 4 \\ 2x_1 + x_2 - 9x_3 + 5x_4 &= -1 \end{aligned}$$

4.) Zjistěte, zda jsou vektory (polynomy) $\mathbf{e}_1 = x^2 - x + 2$, $\mathbf{e}_2 = 3x^2 - x + 3$, $\mathbf{e}_3 = x^2 + 2x + 2$ lineárně nezávislé.

5.) Lineární zobrazení $A : \mathbb{R}^3 \mapsto \mathbb{R}^2$ je dáno hodnotami $A((1,-1,2)) = (0,1)$, $A((0,1,1)) = (2,0)$, $A((0,2,1)) = (2,1)$. Určete nulový prostor (jádro) tohoto lineárního zobrazení, jeho bázi a dimenzi.

$\boxed{A}$

1.)
$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ -2 & -3 & 2 & 0 & 1 & 0 \\ 1 & 5 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ +2r_1 \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 3 & 0 & -1 & 0 & 1 \end{array}\right)\begin{array}{l} -2r_2 \\ \\ -3r_2 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -4 & -3 & -2 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & -6 & -7 & -3 & 1 \end{array}\right)\begin{array}{l} \cdot 6 \\ \cdot 6 \\ \cdot(-1) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 6 & 0 & -24 & -18 & -12 & 0 \\ 0 & 6 & 12 & 12 & 6 & 0 \\ 0 & 0 & 6 & 7 & 3 & -1 \end{array}\right)\begin{array}{l} +4r_3 \\ -2r_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 6 & 0 & 0 & 10 & 0 & -4 \\ 0 & 6 & 0 & -2 & 0 & 2 \\ 0 & 0 & 6 & 7 & 3 & -1 \end{array}\right) \quad \Rightarrow A^{-1} = \frac{1}{6} \cdot \begin{pmatrix} 10 & 0 & -4 \\ -2 & 0 & 2 \\ 7 & 3 & -1 \end{pmatrix} \quad \checkmark$$

2.)
$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 3 & -2 & 1 & 1 & 3 \\ -1 & 0 & 3 & 1 & 3 \\ 1 & -2 & 7 & 3 & 9 \end{array}\right)\begin{array}{l} \\ -3r_1 \\ +r_1 \\ -r_1 \end{array} \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -5 & -2 & -6 \\ 0 & -1 & 5 & 2 & 6 \\ 0 & -1 & 5 & 2 & 6 \end{array}\right)\begin{array}{l} \\ \\ +r_2 \\ +r_2 \end{array} \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -5 & -2 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_2 - 5x_3 - 2x_4 = -6 \quad / x_3 = s,\ x_4 = t$

$x_2 = -6 + 5s + 2t$

$x_1 - (-6 + 5s + 2t) + 2s + t = 3 \Rightarrow x_1 = -3 + 3s + t$

$$\Rightarrow \vec{X} = \begin{pmatrix} -3 + 3s + t \\ -6 + 5s + 2t \\ s \\ t \end{pmatrix};\ s, t \in \mathbb{R} \quad \checkmark$$

3.)
$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 1 & 2 & -1 & -4 \\ 0 & 1 & 5 & 3 \end{array}\right)\begin{array}{l} \\ -r_1 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -3 \\ 0 & 1 & 5 & 3 \end{array}\right)\begin{array}{l} \\ \\ -r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 6 & 6 \end{array}\right)\left.\begin{array}{l} \Rightarrow k_1 = 1 \\ \Rightarrow k_2 = -2 \\ \Rightarrow k_3 = 1 \end{array}\right\} \Rightarrow A((-1,-4,3)) = 1(1,-1) - 2(0,1) + 1(3,2) =$$

$= (4,-1) \quad \checkmark$

4.)
$$\left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 2 & -1 & 1 & 5 \\ 1 & 0 & -1 & 0 \end{array}\right)\begin{array}{l} \\ +2r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 0 & -3 & 1 & -1 \\ 0 & -1 & -1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 0 & -1 & -1 & -3 \\ 0 & -3 & 1 & -1 \end{array}\right)\begin{array}{l} \\ \\ -3r_2 \end{array} \sim \left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right)\begin{array}{l} \Rightarrow k_1 = 2 \\ \Rightarrow k_2 = 1 \\ \Rightarrow k_3 = 2 \end{array}$$

$$\Rightarrow (-3x^2 + 5x)_{\langle E \rangle} = (2, 1, 2) \quad \checkmark$$

5.)
$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & 5 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ 4 & -1 & -1 \\ 10 & -5 & -2 \end{pmatrix} \quad \checkmark$$

$\boxed{B}$

1.) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 2 & 3 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 5 & 9 \\ 1 & -1 & -7 \\ 2 & 4 & 6 \end{pmatrix}}}$ ✓

2.) $\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ -3r_1 \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & -3 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right)\begin{array}{l} \cdot 3 \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 3 & 3 & 6 & 3 & 0 & 0 \\ 0 & -3 & -4 & -3 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right)\begin{array}{l} +r_2 \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 0 & 1 & 0 \\ 0 & -3 & -4 & -3 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right)\begin{array}{l} +r_3 \\ -2r_3 \\ \\ \end{array} \sim$

$\left(\begin{array}{ccc|ccc} 3 & 0 & 0 & -1 & 1 & 1 \\ 0 & -3 & 0 & -1 & 1 & -2 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right)\begin{array}{l} \cdot 2 \\ \cdot(-2) \\ \cdot(-3) \end{array} \sim \left(\begin{array}{ccc|ccc} 6 & 0 & 0 & -2 & 2 & 2 \\ 0 & 6 & 0 & 2 & -2 & 4 \\ 0 & 0 & 6 & 3 & 0 & -3 \end{array}\right) \Rightarrow \underline{\underline{A^{-1} = \frac{1}{6}\begin{pmatrix} -2 & 2 & 2 \\ 2 & -2 & 4 \\ 3 & 0 & -3 \end{pmatrix}}}$ ✓

3.) $\left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 3 \\ 1 & 2 & 3 & 4 & 10 \\ 1 & 0 & 1 & 2 & 4 \\ 2 & 1 & -9 & 5 & -1 \end{array}\right)\begin{array}{l} \\ -r_1 \\ -r_1 \\ -2r_1 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 3 \\ 0 & 1 & 5 & 1 & 7 \\ 0 & -1 & 3 & -1 & 1 \\ 0 & -1 & -5 & -1 & -7 \end{array}\right)\begin{array}{l} \\ \\ +r_2 \\ +r_2 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 3 \\ 0 & 1 & 5 & 1 & 7 \\ 0 & 0 & 8 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \\ \Rightarrow x_2 + 5 + \underset{=s}{x_4} = 7 \\ \Rightarrow \underline{x_3 = 1} \\ \\ \end{array}$

$\underline{x_4 = s} \Rightarrow \underline{x_2 = 2 - s}$ $\qquad x_1 + (2-s) - 2\cdot 1 + 3s = 3 \Rightarrow$
$\underline{x_1 = 3 - 2s}$

$\underline{\underline{\bar{X} = \begin{pmatrix} 3-2s \\ 2-s \\ 1 \\ s \end{pmatrix}, s \in \mathbb{R}}}$

4.) $\begin{pmatrix} 1 & -1 & 2 \\ 3 & -1 & 3 \\ 1 & 2 & 2 \end{pmatrix}\begin{array}{l} \\ -3r_1 \\ -r_1 \end{array} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 0 & 3 & 0 \end{pmatrix}\begin{array}{l} \\ \\ :(3) \end{array} \updownarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & -3 \end{pmatrix}\begin{array}{l} \\ \\ -2r_2 \end{array} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \Rightarrow$ <u>Jsou lin. nezávislé</u>

5.)
$(1,-1,2) \to (0,1)$
$(0,1,1) \to (2,0)$
$\underline{(0,2,1)} \to \underline{(2,1)}$
$\quad ? \qquad\quad (0,0)$

$\left(\begin{array}{ccc|c} 0 & 2 & 2 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \updownarrow \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right) \Rightarrow 2a_2 + 2a_3 = 0 \qquad | a_3 = s$
$\Downarrow$
$a_2 = -s$

$a_1 + s = 0 \Rightarrow a_1 = -s$

$\Rightarrow \bar{x} = -s(1,-1,2) - s(0,1,1) + s(0,2,1) = (-s, 2s, -2s)$

$\Rightarrow \underline{\underline{N(A) = \{ s(-1,2,-2) \mid s \in \mathbb{R} \}}}$

$\underline{\underline{\text{Báze } N(A): \{(-1,2,-2)\}}}$

$\underline{\underline{\dim N(A) = 1}}$ ✓