Gauss - Jordanova metoda

Řešíme soustavu rovnic $\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right)$ převedeme na $\left(\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & x_1 \\ 0 & 1 & \cdots & 0 & x_2 \\ \vdots & & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & x_n \end{array}\right)$

(Používáme u čtvercových matic A)

Př: Gauss-Jordanovou metodou vyřešte soustavu rovnic:

$$x_1 - 2x_2 + x_3 = -1$$
$$-x_1 + x_2 - 3x_3 = -1$$
$$2x_1 + 3x_2 - x_3 = 12$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ -1 & 1 & -3 & -1 \\ 2 & 3 & -1 & 12 \end{array}\right)\begin{array}{l} \\ +r_1 \\ -2r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & -1 & -2 & -2 \\ 0 & 7 & -3 & 14 \end{array}\right)\begin{array}{l} -2r_2 \\ \\ +7r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 5 & 3 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & -17 & 0 \end{array}\right)\begin{array}{l} \\ \cdot(-1) \\ :(-17) \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 5 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right)\begin{array}{l} -5r_3 \\ -2r_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{array}{l} \underline{\underline{x_1 = 3}} \\ \underline{\underline{x_2 = 2}} \\ \underline{\underline{x_3 = 0}} \end{array} \Rightarrow \underline{\underline{\bar{x} = (3, 2, 0)}}$$

Př: Gauss-Jordanovou metodou vyřešte soustavu rovnic:

$$\begin{array}{l} 2x_1 - x_2 + 3x_3 = 5 \\ -3x_1 + x_2 - x_3 = -4 \\ 4x_1 + 3x_2 - x_3 = 3 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 2 & -1 & 3 & 5 \\ -3 & 1 & -1 & -4 \\ 4 & 3 & -1 & 3 \end{array}\right)\begin{array}{l} \\ \cdot 2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 5 \\ -6 & 2 & -2 & -8 \\ 4 & 3 & -1 & 3 \end{array}\right)\begin{array}{l} \\ +3r_1 \\ -2r_1 \end{array} \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 5 \\ 0 & -1 & 7 & 7 \\ 0 & 5 & -7 & -7 \end{array}\right)\begin{array}{l} +r_2 \\ \\ +5r_2 \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 2 & 0 & -4 & -2 \\ 0 & -1 & 7 & 7 \\ 0 & 0 & 28 & 28 \end{array}\right)\begin{array}{l} :(2) \\ \cdot(-1) \\ :(28) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -1 \\ 0 & 1 & -7 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right)\begin{array}{l} +2r_3 \\ +7r_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{array}{l} \underline{\underline{x_1 = 1}} \\ \underline{\underline{x_2 = 0}} \\ \underline{\underline{x_3 = 1}} \end{array} \Rightarrow$$

$$\underline{\underline{\bar{x} = (1, 0, 1)}}$$

Př. Gauss-Jordanovou metodou vyřešíme soustavu:

$$
\begin{aligned}
3x_1 - 2x_2 + 2x_3 &= 10 \\
x_1 + 3x_2 - x_3 &= 2 \\
2x_1 + 2x_2 + 3x_3 &= 15
\end{aligned}
$$

$$
\left(\begin{array}{ccc|c} 3 & -2 & 2 & 10 \\ 1 & 3 & -1 & 2 \\ 2 & 2 & 3 & 15 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 3 & -2 & 2 & 10 \\ 2 & 2 & 3 & 15 \end{array}\right) \begin{array}{l} \cdot(-3) \quad / \cdot(-2) \\ + \\ + \end{array} \sim
$$

$$
\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & -11 & 5 & 4 \\ 0 & -4 & 5 & 11 \end{array}\right) \begin{array}{l} \\ \cdot(-4) \\ \cdot(11) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & -11 & 5 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 44 & -20 & -16 \\ 0 & -44 & 55 & 121 \end{array}\right) \begin{array}{l} \\ \\ + \end{array} \sim
$$

$$
\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 44 & -20 & -16 \\ 0 & 0 & 35 & 105 \end{array}\right) \begin{array}{l} \\ :4 \\ :35 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} + \\ + \\ \cdot 5 \end{array}
$$

Nyní dostaneme nuly i nad diagonálu (opíšeme 3. řádek a upravíme 1. a 2. řádek)

$$
\sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 11 & 0 & 11 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} \\ :11 \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} + \\ \cdot(-3) \\ \end{array} \sim
$$

$$
\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right) \Rightarrow \begin{aligned} x_1 &= 2 \\ x_2 &= 1 \\ x_3 &= 3 \end{aligned} \Rightarrow \bar{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = (2, 1, 3)^T
$$

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Př: Gauss-Jordanovou metodou vyřešíme soustavu

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 3 \\ 3x_1 + x_2 - x_3 &= 7 \\ -2x_1 + x_2 - 2x_3 &= -3 \end{aligned} \quad \Rightarrow$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 3 & 1 & -1 & 7 \\ -2 & 1 & -2 & -3 \end{array}\right) \begin{array}{c} \\ \cdot 2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 6 & 2 & -2 & 14 \\ -2 & 1 & -2 & -3 \end{array}\right) \begin{array}{c} \\ -3r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 3 \\ 0 & 5 & -11 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{c} -3r_3 \\ +11r_3 \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 2 & -1 & 0 & 3 \\ 0 & 5 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{c} \\ :5 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & -1 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{c} +r_2 \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} \Rightarrow x_1 = 2 \\ \Rightarrow x_2 = 1 \\ \Rightarrow x_3 = 0 \end{array}$$

$$\Rightarrow \quad \bar{x} = (2, 1, 0)$$

Př. Gauss-Jordanovou metodou vyřešte soustavy lineárních rovnic:

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ -x_1 + 2x_2 - x_3 &= 0 \\ -x_1 + x_2 - 2x_3 &= -2 \end{aligned} \qquad \begin{aligned} x_1 - x_2 + x_3 &= 3 \\ -x_1 + 2x_2 - x_3 &= -3 \\ -x_1 + x_2 - 2x_3 &= -5 \end{aligned} \qquad \begin{aligned} x_1 - x_2 + x_3 &= 1 \\ -x_1 + 2x_2 - x_3 &= 4 \\ -x_1 + x_2 - 2x_3 &= -2 \end{aligned}$$

Všechny tři soustavy mají stejnou matici soustavy, liší se jen pravými stranami. (Jde o tzv. soustavu s více pravými stranami.) $\Rightarrow$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 3 & 1 \\ -1 & 2 & -1 & 0 & -3 & 4 \\ -1 & 1 & -2 & -2 & -5 & -2 \end{array}\right) \begin{array}{c} \\ +r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 3 & 1 \\ 0 & 1 & 0 & 1 & 0 & 5 \\ 0 & 0 & -1 & -1 & -2 & -1 \end{array}\right) \begin{array}{c} +r_3 \\ \\ \cdot(-1) \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 & 2 & 1 \end{array}\right) \begin{array}{c} +r_2 \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{c|c} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{E} & \underbrace{\begin{matrix} 1 & 1 & 5 \\ 1 & 0 & 5 \\ 1 & 2 & 1 \end{matrix}}_{\text{řešení}} \end{array}\right) \Rightarrow$$

řešení 1. soustavy: $\bar{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

řešení 2. soustavy: $\bar{x} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

řešení 3. soustavy: $\bar{x} = \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix}$