**A**

1.) Je dána matice $A = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 4 \end{pmatrix}$. Určete matici k ní inverzní.

2.) Vyřešte soustavu lineárních rovnic:

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 0 \\ -x_1 + 2x_2 + 3x_3 + x_4 &= 0 \\ -x_1 + 3x_2 + 7x_3 + x_4 &= 0 \\ -2x_1 + 5x_2 + 10x_3 + 2x_4 &= 0 \end{aligned}$$

3.) Ověřte, zda vektory $\mathbf{e}_1 = (1, -1, 8)$, $\mathbf{e}_2 = (5, -5, 1)$, $\mathbf{e}_3 = (-1, 1, 3)$ tvoří bázi vektorového prostoru $(\mathbb{R}^3, +, \cdot)$.

4.) Určete souřadnice polynomu (vektoru) $3x^2 + 3x + 4$ vzhledem k bázi $E = \{x^2 + 2x + 1,\ x^2 - x, x + 1\}$

5.) Lineární zobrazení $A : \mathbb{R}^3 \mapsto \mathbb{R}^3$ je dáno hodnotami:

$$\begin{aligned} A((1,0,1)) &= (1,1,3) \\ A((1,1,1)) &= (1,1,0) \\ A((0,1,1)) &= (0,0,1) \end{aligned}$$

Určete jádro lineárního zobrazení $A$.

**B**

1.) Je dána matice $A = \begin{pmatrix} 1 & 1 & -1 \\ 4 & 3 & 3 \\ 4 & 4 & 3 \end{pmatrix}$. Určete matici k ní inverzní.

2.) Vyřešte soustavu lineárních rovnic:

$$\begin{aligned} x_1 + 2x_2 + x_3 - x_4 &= 0 \\ x_1 + 4x_2 + 9x_3 - x_4 &= 0 \\ -3x_1 - 5x_2 + x_3 + 3x_4 &= 0 \\ -x_1 - x_2 + 3x_3 + x_4 &= 0 \end{aligned}$$

3.) Zjistěte, zda je vektor $(3, 1, 4)$ lineární kombinací vektorů $(1, 2, 1)$, $(1, 0, 1)$ a $(1, -1, 1)$

4.) Určete souřadnice vektoru $\mathbf{x} = (3, -6, 6)$ vzhledem k bázi $E = \{\mathbf{e}_1 = (-1, 1, 3), \mathbf{e}_2 = (2, 1, 1), \mathbf{e}_3 = (1, -1, 8)\}$.

5.) Lineární zobrazení $A : \mathbb{R}^3 \mapsto \mathbb{R}^3$ je dáno hodnotami:

$$\begin{aligned} A((1, 0, 1)) &= (1, 1, 3) \\ A((1, 1, 1)) &= (1, 1, 0) \\ A((0, 1, 1)) &= (0, 0, 1) \end{aligned}$$

Určete jeho hodnotu $A((-1, 1, 2))$.

$\boxed{A}$

1.) Určete matici inverzní k matici $A = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 4 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 4 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ +r_1 \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 & 0 \\ 0 & -1 & 4 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ +r_2 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right) \begin{array}{l} +3r_3 \\ +3r_3 \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & +0 & 4 & 3 & 3 \\ 0 & 1 & 0 & 4 & 4 & 3 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 4 & 3 & 3 \\ 4 & 4 & 3 \\ 1 & 1 & 1 \end{pmatrix} \quad \checkmark$$

2.) Vyřešte soustavu lineárních rovnic:

$$\left.\begin{array}{r} x_1 - x_2 + x_3 - x_4 = 0 \\ -x_1 + 2x_2 + 3x_3 + x_4 = 0 \\ -x_1 + 3x_2 + 7x_3 + x_4 = 0 \\ -2x_1 + 5x_2 + 10x_3 + 2x_4 = 0 \end{array}\right\} \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ -1 & 2 & 3 & 1 & 0 \\ -1 & 3 & 7 & 1 & 0 \\ -2 & 5 & 10 & 2 & 0 \end{array}\right) \begin{array}{l} \\ +r_1 \\ +r_1 \\ +2r_1 \end{array} \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 2 & 8 & 0 & 0 \\ 0 & 3 & 12 & 0 & 0 \end{array}\right) \begin{array}{l} \\ \\ -2r_2 \\ -3r_2 \end{array} \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 + 4t + t - \overset{s}{x_4} = 0$

$x_3 = t \Rightarrow x_2 = -4t$

$$\Rightarrow (x_1, x_2, x_3, x_4) = \underline{\underline{(-5t + s, -4t, t, s)}} = t(-5,-4,1,0) + s(1,0,0,1) \;;\; s,t \in \mathbb{R} \quad \checkmark$$

3.) Ověřte, zda vektory $\bar{e}_1 = (1,-1,8)$; $\bar{e}_2 = (5,-5,1)$; $\bar{e}_3 = (-1,1,3)$ tvoří bázi vektorového prostoru $(\mathbb{R}^3, +, \cdot)$.

$$\begin{pmatrix} 1 & -1 & 8 \\ 5 & -5 & 1 \\ -1 & 1 & 3 \end{pmatrix} \begin{array}{l} \\ -5r_1 \\ +r_1 \end{array} \sim \begin{pmatrix} 1 & -1 & 8 \\ 0 & 0 & -39 \\ 0 & 0 & 12 \end{pmatrix} \begin{array}{l} \\ :(39) \\ :(12) \end{array} \sim \begin{pmatrix} 1 & -1 & 8 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \\ +r_2 \end{array} \sim \begin{pmatrix} 1 & -1 & 8 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ jsou lin. závislé $\Rightarrow$ <u>netvoří bázi $(\mathbb{R}^3, +, \cdot)$</u> $\checkmark$

4.) Určete souřadnice polynomu $\underset{(3,3,4)}{3X^2 + 3X + 4}$ vzhledem k bázi $E = \{\underset{(1,2,1)}{X^2 + 2X + 1}, \underset{(1,-1,0)}{X^2 - X}, \underset{(0,1,1)}{X + 1}\}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 2 & -1 & 1 & 3 \\ 1 & 0 & 1 & 4 \end{array}\right) \begin{array}{l} \\ -2r_1 \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -3 & 1 & -3 \\ 0 & -1 & 1 & 1 \end{array}\right) \begin{array}{l} \\ \\ r_3 \cdot (-3) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -3 & 1 & -3 \\ 0 & 3 & -3 & -3 \end{array}\right) \begin{array}{l} \\ \\ +r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -3 & 1 & -3 \\ 0 & 0 & -2 & -6 \end{array}\right)$$

$\Rightarrow k_1 + 1 = 3 \Rightarrow k_1 = 1$

$\Rightarrow -3k_2 + 3 = -3 \Rightarrow k_2 = 2$

$\Rightarrow k_3 = 3$

$$\Rightarrow \underline{\underline{(3X^2 + 3X + 4)_{\langle E \rangle} = (1, 2, 3)}} \quad \checkmark$$

5.) Lineární zobrazení je dáno hodnotami $A((1,0,1)) = (1,1,3)$; $A((1,1,1)) = (1,1,0)$; $A((0,1,1)) = (0,0,1)$.
Určete $\operatorname{Ker} A$.

$$\left.\begin{array}{l} (1,0,1) \to (1,1,3) \\ (1,1,1) \to (1,1,0) \\ (0,1,1) \to (0,0,1) \\ \hline \bar{x} \to (0,0,0) \end{array}\right\} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} \\ -r_1 \\ -3r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & -3 & 1 & 0 \end{array}\right)$$

$\Rightarrow \underline{k_1 = -t}$

$\Rightarrow \underline{k_2 = t} \Rightarrow \underline{k_3 = 3t}$ $\Rightarrow$

$$\Rightarrow \bar{x} = -t(1,0,1) + t(1,1,1) + 3t(0,1,1) = (0, 4t, 3t) = t(0,4,3), \; t \in \mathbb{R}$$

$$\Rightarrow \underline{\underline{\operatorname{Ker} A = \{ t(0,4,3) \mid t \in \mathbb{R} \}}} \quad \checkmark$$

$$\underline{\underline{\text{Báze } \operatorname{Ker} A = \{(0,4,3)\}}} \quad \checkmark$$

$$\underline{\underline{\dim \operatorname{Ker} A = 1}} \quad \checkmark$$

B

1.) Určete matici inverzní k matici $\begin{pmatrix} 1 & 1 & -1 \\ 4 & 3 & 3 \\ 4 & 4 & 3 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 4 & 3 & 3 & 0 & 1 & 0 \\ 4 & 4 & 3 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ -4r_1 \\ -4r_1 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 7 & -4 & 1 & 0 \\ 0 & 0 & 7 & -4 & 0 & 1 \end{array}\right)\begin{array}{l} \cdot 7 \\ -r_2 \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 7 & 7 & -7 & 7 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 7 & -4 & 0 & 1 \end{array}\right)\begin{array}{l} +r_3 \\ (-1) \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 7 & 7 & 0 & 3 & 0 & 1 \\ 0 & 7 & 0 & 0 & -7 & 7 \\ 0 & 0 & 7 & -4 & 0 & 1 \end{array}\right)\begin{array}{l} -r_2 \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 7 & 0 & 0 & 3 & 7 & -6 \\ 0 & 7 & 0 & 0 & -7 & 7 \\ 0 & 0 & 7 & -4 & 0 & 1 \end{array}\right) \Rightarrow A^{-1} = \frac{1}{7}\begin{pmatrix} 3 & 7 & -6 \\ 0 & -7 & 7 \\ -4 & 0 & 1 \end{pmatrix}$$

2.) Vyřešte soustavu lin. rovnic:

$$\left.\begin{array}{r} x_1 + 2x_2 + x_3 - x_4 = 0 \\ x_1 + 4x_2 + 9x_3 - x_4 = 0 \\ -3x_1 - 5x_2 + x_3 + 3x_4 = 0 \\ -x_1 - x_2 + 3x_3 + x_4 = 0 \end{array}\right\} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 1 & 4 & 9 & -1 & 0 \\ -3 & -5 & 1 & 3 & 0 \\ -1 & -1 & 3 & 1 & 0 \end{array}\right)\begin{array}{l} \\ -r_1 \\ +3r_1 \\ +r_1 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & 2 & 8 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow x_1 - 8t + t - \overset{s}{x_4} = 0$

$\Rightarrow x_3 = t \Rightarrow x_2 = -4t$

$$\Rightarrow \bar{x} = \begin{pmatrix} 7t + s \\ -4t \\ t \\ s \end{pmatrix} = t\begin{pmatrix} 7 \\ -4 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \; s, t \in \mathbb{R}$$

3.) Zjistěte, zda je vektor $(3,1,4)$ lineární kombinací vektorů $(1,2,1)$; $(1,0,1)$; $(1,-1,1)$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 0 & -1 & 1 \\ 1 & 1 & 1 & 4 \end{array}\right)\begin{array}{l} \\ -2r_1 \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -5 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow 0 = 1$$

$\Rightarrow$ soust. nemá řešení $\Rightarrow$ není jejich lin. kombinací

4.) Určete souřadnice vektoru $\bar{x} = (3,-6,6)$ vzhledem k bázi $E = \{(-1,1,3); (2,1,1); (1,-1,8)\}$.

$$\left(\begin{array}{ccc|c} -1 & 2 & 1 & 3 \\ 1 & 1 & -1 & -6 \\ 3 & 1 & 8 & 6 \end{array}\right)\begin{array}{l} \\ +r_1 \\ +3r_1 \end{array} \sim \left(\begin{array}{ccc|c} -1 & 2 & 1 & 3 \\ 0 & 3 & 0 & -3 \\ 0 & 7 & 11 & 15 \end{array}\right) \Rightarrow k_2 = -1 \Rightarrow 7(-1) + 11k_3 = 15 \Rightarrow k_3 = 2 \Rightarrow -k_1 + 2 + 2 = 3 \Rightarrow k_1 = 3$$

$$\Rightarrow (3,-6,6)_{\langle E \rangle} = (-3,-1,2)$$

5.) Určete $A((-1,1,2))$, jestliže jsou dány hodnoty lineárního zobrazení $A$:

$$\left.\begin{array}{l} (1,0,1) \longrightarrow (1,1,3) \\ (1,1,1) \longrightarrow (1,1,0) \\ (0,1,1) \longrightarrow (0,0,1) \\ \hline (-1,1,2) \longrightarrow \; ? \end{array}\right\} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{array}\right)\begin{array}{l} \\ \\ -r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} \Rightarrow k_1 - 2 = -1 \Rightarrow k_1 = 1 \\ \Rightarrow k_2 + 3 = 1 \Rightarrow k_2 = -2 \\ \Rightarrow k_3 = 3 \end{array}$$

$$\Rightarrow A((-1,1,2)) = 1(1,1,3) - 2(1,1,0) + 3(0,0,1) = (-1,-1,6)$$