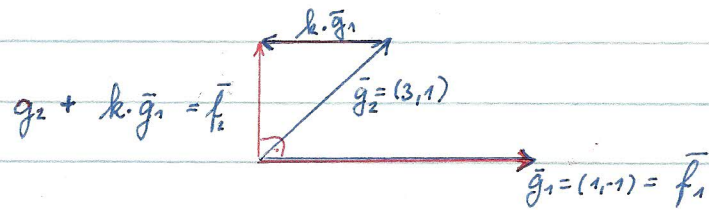


Gramm - Schmidtův ortogonalizační proces v \mathbb{R}^2

Pr. min: Ortogonalizujte bázi $G = \{ (1, -1); (3, 1) \}$ a poté barové vektory normalizujte (tzn. ortonormalizujte bázi G).



$$1.) \bar{f}_1 = \bar{g}_1 = (1, -1)$$

$$2.) \bar{f}_2 = \bar{g}_2 + k \bar{f}_1 = (3, 1) + k(1, -1) \quad \text{a musí platit: } \bar{f}_1 \cdot \bar{f}_2 = 0 \Rightarrow$$

$$(1, -1) [(3, 1) + k(1, -1)] = 0$$

$$2 + k \cdot 2 = 0$$

$$k = -1 \Rightarrow \bar{f}_2 = (3, 1) - 1 \cdot (1, -1) = (2, 2)$$

\Rightarrow Ortogonální báze $F = \{ (1, -1); (2, 2) \}$

3.) Normalizace:

$$\bar{e}_1 = \frac{1}{\sqrt{\bar{f}_1 \cdot \bar{f}_1}} \bar{f}_1 = \frac{1}{\sqrt{(1, -1) \cdot (1, -1)}} (1, -1) = \frac{1}{\sqrt{2}} (1, -1)$$

$$\bar{e}_2 = \frac{1}{\sqrt{\bar{f}_2 \cdot \bar{f}_2}} \bar{f}_2 = \frac{1}{\sqrt{(2, 2) \cdot (2, 2)}} = \frac{1}{\sqrt{8}} (2, 2)$$

\Rightarrow Orto normální báze: $E = \left\{ \frac{1}{\sqrt{2}} (1, -1); \frac{1}{\sqrt{8}} (2, 2) \right\}$