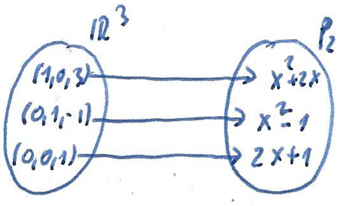
Př. Lineární zobrazení $A : \mathbb{R}^3 \to P_2$ je dáno hodnotami:

$A((1,0,3)) = x^2+2x \quad , \quad A((0,1,-1)) = x^2-1 \quad , \quad A((0,0,1)) = 2x+1$



$P_2 = \{ ax^2+bx+c \mid a,b,c \in \mathbb{R} \}$

a) Určete $A(1,-1,6)$.

$k_1(1,0,3) + k_2(0,1,-1) + k_3(0,0,1) = (1,-1,6)$

$k_1(x^2+2x) + k_2(x^2-1) + k_3(2x+1) = A(1,-1,6)$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 3 & -1 & 1 & 6 \end{array}\right) \begin{array}{c} \\ \\ -3r_1 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 3 \end{array}\right) \begin{array}{c} \\ \\ +r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} \Rightarrow k_1 = 1 \\ \Rightarrow k_2 = -1 \\ \Rightarrow k_3 = 2 \end{array} \Bigg\} \Rightarrow$$

$A(1,-1,6) = 1(x^2+2x) - 1(x^2-1) + 2(2x+1) = \underline{\underline{6x+3}}$

b) Určete $\bar{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$ tak, aby $A(\bar{x}) = 6x+3$

$k_1(1,0,3) + k_2(0,1,-1) + k_3(0,0,1) = \bar{x}$

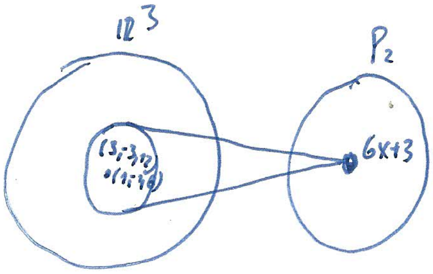
$k_1(x^2+2x) + k_2(x^2-1) + k_3(2x+1) = A(\bar{x}) = 6x+3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 6 \\ 0 & -1 & 1 & 3 \end{array}\right) \begin{array}{c} \\ -2r_1 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 2 & 6 \\ 0 & -1 & 1 & 3 \end{array}\right) \begin{array}{c} \\ :2 \\ -\frac{1}{2}r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \Rightarrow k_1 + t - 3 = 0 \quad \Rightarrow k_1 = 3-t \\ \Rightarrow -k_2 + k_3 = 3 \quad / k_3 = t / \Rightarrow k_2 = t-3 \\ \\ \end{array}$$

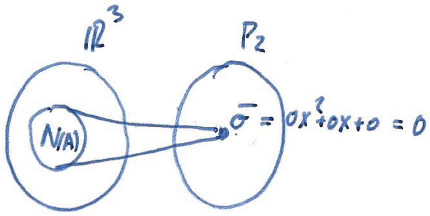
$\Rightarrow \bar{x} = (3-t)(1,0,3) + (t-3)(0,1,-1) + t(0,0,1) =$

$= (3-t, 0, 9-3t) + (0, t-3, 3-t) + (0,0,t) =$

$= \underline{\underline{(3-t, t-3, 12-3t), \ t \in \mathbb{R}}}$ (zvolíme-li $t = 2 \Rightarrow \bar{x} = (1,-1,6)$)



c) Určete jádro lin. zobr. A (= nulový prostor lin. zobr. A = N(A)), jeho bázi a dimenzi N(A) (= dim N(A) = d(A) = defekt A).



$\Rightarrow N(A) = \operatorname{Ker} A = \{\bar{x} \in \mathbb{R}^3 \mid A(\bar{x}) = 0\}$

$k_1(1,0,3) + k_2(0,1,-1) + k_3(0,0,1) = \bar{x}$

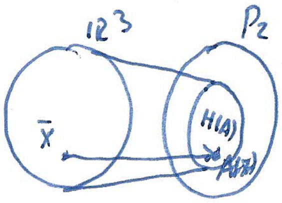
$k_1(x^2+2x) + k_2(x^2-1) + k_3(2x+1) = A(\bar{x}) = 0x^2+0x+0 = 0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) -2r_1 \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \begin{array}{l} \\ :(-2) \\ -\frac{1}{2}r_2 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \Rightarrow k_1 + t = 0 \Rightarrow k_1 = -t \\ \Rightarrow -k_2 + k_3 = 0 \quad |k_3 = t| \Rightarrow k_2 = t \end{array}$$

$\Rightarrow \bar{x} = -t(1,0,3) + t(0,1,-1) + t(0,0,1) = (-t, t, -3t) = t(-1,1,-3)$

$\Rightarrow \underline{\underline{N(A) = \{t(-1,1,-3) \mid t \in \mathbb{R}\}}} \Rightarrow \underline{\underline{\text{Báze } N(A) = \{(-1,1,-3)\}}} \Rightarrow \underline{\underline{d(A) = \dim N(A) = 1}}$

d) Určete obor hodnot zobrazení A (= H(A)), jeho bázi a jeho dimenzi (dim H(A) = h(A) = hodnost A)



$H(A) = \{A(\bar{x}) \mid \bar{x} \in \mathbb{R}^3\}$

$(k_1(1,0,3) + k_2(0,1,-1) + k_3(0,0,1) = \bar{x} \Rightarrow k_1(x^2+2x) + k_2(x^2-1) + k_3(2x+1) = A(\bar{x}))$

$\Rightarrow H(A) = \{k_1(x^2+2x) + k_2(x^2-1) + k_3(2x+1) \mid k_1, k_2, k_3 \in \mathbb{R}\}$

Tvoří x^2+2x, x^2-1 a $2x+1$ bázi H(A)? Musíme ověřit nezávislost! :

$$\underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 2 & 1 \end{pmatrix} -r_1 \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{pmatrix} +r_2 \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{upravit na schodový tvar}} \Rightarrow$$

jsou lin. závislé! ⇒ za bázi vezmeme polynomy reprezentované nenulovými řádky ⇒

$\Rightarrow \underline{\underline{\text{Báze } H(A) = \{x^2+2x, -2x-1\}}} \Rightarrow \underline{\underline{H(A) = \{k_1(x^2+2x) + k_2(-2x-1) \mid k_1, k_2 \in \mathbb{R}\}}}$

$\Downarrow$

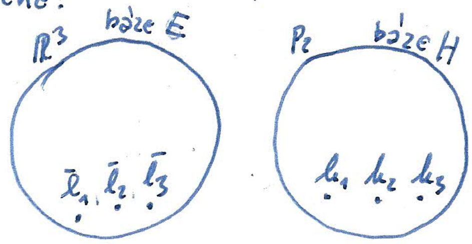
$\underline{\underline{\dim H(A) = \underline{2} = h(A)}}$

$A: U \to V \Rightarrow$

Pozn.: Platí $d(A) + h(A) = \dim U$ a opravdu $\dim N(A) + \dim H(A) = 1 + 2 = 3 = \dim \mathbb{R}^3$

e) Určete matici lin. zobrazení A vzhledem ke standardním bázím $S_1 = \{\underset{\bar{e}_1}{\underset{\shortparallel}{(1,0,0)}};\ \underset{\bar{e}_2}{\underset{\shortparallel}{(0,1,0)}};\ \underset{\bar{e}_3}{\underset{\shortparallel}{(0,0,1)}}\}$ a $S_2=\{\underset{\bar{h}_1}{\underset{\shortparallel}{x^2}}, \underset{\bar{h}_2}{\underset{\shortparallel}{x}}, \underset{\bar{h}_3}{\underset{\shortparallel}{1}}\}$

Obecně:



$$\Rightarrow A_{EH} = \begin{pmatrix} A(\bar{e}_1)_{\langle H\rangle} & A(\bar{e}_2)_{\langle H\rangle} & A(\bar{e}_3)_{\langle H\rangle} \end{pmatrix}$$

Vzhledem ke stand. bázím je to jednoduché:

$$\begin{array}{cc} \bar{x} \longrightarrow A(\bar{x}) \end{array}$$

$$\underbrace{\begin{pmatrix}1&0&3\\0&1&-1\\0&0&1\end{pmatrix}}_{\text{upravíme na matici jednotkovou}}\!\!\begin{matrix}|\\|\\|\end{matrix}\begin{pmatrix}1&2&0\\1&0&-1\\0&2&1\end{pmatrix}\begin{matrix}-3r_3\\+r_3\\ \end{matrix} \sim \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\!\!\begin{matrix}|\\|\\|\end{matrix}\underbrace{\begin{pmatrix}1&-4&-3\\1&2&0\\0&2&1\end{pmatrix}}_{\text{stačí transponovat tuto matici a obdržíme matici lin. zobr. A vzhledem ke st. bázím.}}$$

$A((1,0,0)) = x^2-4x-3 \quad \Rightarrow A(\bar{e}_1)_{\langle S_2\rangle} = (1,-4,-3)$

$\Rightarrow A((0,1,0)) = x^2+2x \quad \Rightarrow A(\bar{e}_2)_{\langle S_2\rangle} = (0,1,0)$

$A((0,0,1)) = 2x+1 \quad \Rightarrow A(\bar{e}_3)_{\langle S_2\rangle} = (0,2,1)$

$$\Rightarrow \quad \underline{\underline{A_{S_1S_2} = \begin{pmatrix}1&1&0\\-4&2&2\\-3&0&1\end{pmatrix}}}$$

f) Určete matici A_{EH}, kde $E=\{(2,0,1);(0,1,1),(0,1,3)\}$, $H=\{x^2, x-1, x\ 1\}$

$$A_{EH} = \underset{\substack{\uparrow\\ \text{H má ve sloupcích}\\ \text{souřadnice}\\ \text{vektorů z H}\\ \text{vzhledem k } S_2}}{H^{-1}} \cdot A_{S_1S_2} \cdot \underset{\substack{\uparrow\\ \text{ve sloupcích}\\ \text{souř. vektorů}\\ \text{z B vzhledem}\\ \text{k } S_1}}{E} = \begin{pmatrix}1&0&0\\0&1&1\\0&-1&0\end{pmatrix}^{-1} \cdot \begin{pmatrix}1&1&0\\-4&2&2\\-3&0&1\end{pmatrix}\begin{pmatrix}2&0&0\\0&1&1\\1&1&3\end{pmatrix}$$

určíme H^{-1}:

$$\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&1&0&1&0\\0&-1&0&0&0&1\end{array}\right)\begin{matrix}\\ \\+r_2\end{matrix} \sim \left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&1&0&1&0\\0&0&1&0&1&1\end{array}\right)\begin{matrix}\\-r_3\\ \end{matrix} \sim \left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&0&0&-1\\0&0&1&0&1&1\end{array}\right) \Rightarrow$$

$$\underbrace{\begin{matrix}1&0&0\\0&0&-1\\0&1&1\end{matrix}}_{H^{-1}}$$

$$A_{EH} = \begin{pmatrix}1&0&0\\0&0&-1\\0&1&1\end{pmatrix}\begin{pmatrix}1&1&0\\-4&2&2\\-3&0&1\end{pmatrix}\begin{pmatrix}2&0&0\\0&1&1\\1&1&3\end{pmatrix} = \begin{pmatrix}1&1&0\\3&0&-1\\-7&2&3\end{pmatrix}\begin{pmatrix}2&0&0\\0&1&1\\1&1&3\end{pmatrix} = \underline{\underline{\begin{pmatrix}2&1&1\\5&-1&-3\\-11&5&11\end{pmatrix}}}$$

Platí:

$$\underset{\substack{\uparrow\\ \text{sloupec}}}{A(\bar{x})_{\langle H\rangle}} = A_{EH} \cdot \overset{\substack{\text{sloupec}\\ \downarrow}}{\bar{x}_{\langle E\rangle}}$$

g) Určete $A((2,0,1))$

α) Pomocí $A_{S_1S_2}$:

$$A((2,0,1))_{S_2} = A_{S_1S_2}\begin{pmatrix}(2,0,1)_{\langle S_1\rangle}\end{pmatrix} = \begin{pmatrix}1&1&0\\-4&2&2\\-3&0&1\end{pmatrix}\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}2\\-6\\-5\end{pmatrix} \Rightarrow \underline{\underline{A((2,0,1)) = 2x^2-6x-5}}$$

nebo:

β) Pomocí A_{EH}:

$$A((2,0,1))_H = A_{EH}\begin{pmatrix}\underset{\substack{\shortparallel\\ (1,0,0)\\ \text{protože}\\ (2,0,1)=1(2,0,1)+0(0,1,1)+0(0,1,3)}}{(2,0,1)_{\langle E\rangle}}\end{pmatrix} = \begin{pmatrix}2&1&1\\5&-1&-3\\-11&5&11\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\5\\-11\end{pmatrix} \Rightarrow A((2,0,1)) = 2(x^2)+5(x-1)-11.(x) =$$

$$\underline{\underline{= 2x^2-6x-5}}$$

Př: Lineární zobrazení $A: \mathbb{R}^3 \to \mathbb{R}^2$ je dáno hodnotami

$A((1,0,3)) = (1,2)$; $A((0,1,-1)) = (3,1)$; $A((1,1,1)) = (0,1)$

a) Určete $A((3,3,4))$

$$(3,3,4) = k_1(1,0,3) + k_2(0,1,-1) + k_3(1,1,1)$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$A((3,3,4)) = k_1(1,2) + k_2(3,1) + k_3(0,1)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 3 & -1 & 1 & 4 \end{array}\right)_{-3r_1} \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & -2 & -5 \end{array}\right)_{+r_2} \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right) \begin{array}{l} \Rightarrow k_1 + 2 = 3 \Rightarrow \underline{k_1 = 1} \\ \Rightarrow k_2 + 2 = 3 \Rightarrow \underline{k_2 = 1} \\ \Rightarrow \underline{k_3 = 2} \end{array} \Rightarrow$$

$$A((3,3,4)) = 1\cdot(1,2) + 1\cdot(3,1) + 2(0,1) = \underline{\underline{(4,5)}}$$

b) Určete $\bar{x} \in \mathbb{R}^3$ tak, aby $A(\bar{x}) = (3,2)$

(určitě to podle zadání bude např. $(0,1,-1) + (1,1,1) = (1,2,0)$)

$$\bar{x} = k_1(1,0,3) + k_2(0,1,-1) + k_3(1,1,1)$$
$$\downarrow$$
$$(3,2) = k_1(1,2) + k_2(3,1) + k_3(0,1)$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right)_{-2r_1} \sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & -5 & 1 & -4 \end{array}\right) \begin{array}{l} k_1 + 3t = 3 \qquad \Rightarrow \underline{k_1 = -3t+3} \\ \Rightarrow -5k_2 + k_3 = -4 \Rightarrow \underline{k_2 = t} \Rightarrow \underline{k_3 = -4+5t} \end{array}$$

$$\bar{x} = (-3t+3)(1,0,3) + t(0,1,-1) + (-4+5t)(1,1,1) = (-3t+3-4+5t,\ t-4+5t,\ -9t+9-t-4+5t) = \underline{\underline{(-1+2t, -4+6t, 5-5t)}}, t \in \mathbb{R}$$

c) Určete jádro lin. zobrazení A, jeho bázi a dimenzi

$$\bar{x} = k_1(1,0,3) + k_2(0,1,-1) + k_3(1,1,1)$$
$$\downarrow$$
$$(0,0) = k_1(1,2) + k_2(3,1) + k_3(0,1)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right)_{-2r_1} \sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & -5 & 1 & 0 \end{array}\right) \begin{array}{l} \Rightarrow k_1 = -3t \\ \Rightarrow k_2 = t \\ \Rightarrow k_3 = 5t \end{array}$$

$$\Rightarrow \bar{x} = -3t(1,0,3) + t(0,1,-1) + 5t(1,1,1) = (2t, 6t, -5t),\ t \in \mathbb{R}$$

$\Rightarrow$ $\underline{\underline{\operatorname{Ker} A = \{t(2,6,-5) \mid t \in \mathbb{R}\}}}$, $\underline{\underline{\text{báze } \operatorname{Ker} A = \{(2,6,-5)\}}}$, $\underline{\underline{\text{dimenze } \operatorname{Ker} A = 1}}$

d) Určete obor hodnot H_A, jeho bázi a dimenzi

$$\left.\begin{array}{l} x = k_1(1,0,3) + k_2(0,1,-1) + k_3(1,1,1) \\ \downarrow \\ A(x) = k_1(1,2) + k_2(3,1) + k_3(0,1) \end{array}\right\} \Rightarrow H_A = \{k_1(1,2) + k_2(3,1) + k_3(0,1) \mid k_1, k_2, k_3 \in \mathbb{R}\} \Rightarrow \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 \\ 0 & -5 \\ 0 & 0 \end{pmatrix}$$

$$\underline{\underline{H_A = \{k_1(1,2) + k_2(0,-5) \mid k_1, k_2 \in \mathbb{R}\}}}$$

$\underline{\underline{\text{báze } H_A = \{(1,2); (0,-5)\}}}$, $\underline{\underline{\dim H_A = 2}}$ $(\Rightarrow H_A = \mathbb{R}^2)$

Př: lin. zobrazení $f : \mathbb{R}^3 \to \mathbb{R}^2$ je dáno hodnotami

$f((1,1,0)) = (0,2)$; $f((1,0,1)) = (1,1)$; $f((0,0,1)) = (-1,2)$

1.) Určete matici lin. zobrazení f (vzhledem ke stand. bázím S_1 a S_2)

$$\begin{array}{c|c} \bar{x} & f(\bar{x}) \\ \left(\begin{matrix} 1&1&0 \\ 1&0&1 \\ 0&0&1 \end{matrix}\right. & \left.\begin{matrix} 0&2 \\ 1&1 \\ -1&2 \end{matrix}\right) \end{array} \begin{matrix} \\ -r_1 \\ \\ \end{matrix} \sim \left(\begin{matrix} 1&1&0 \\ 0&-1&1 \\ 0&0&1 \end{matrix}\middle|\begin{matrix} 0&2 \\ 1&-1 \\ -1&2 \end{matrix}\right) \begin{matrix} \\ -r_3 \\ \\ \end{matrix} \sim \left(\begin{matrix} 1&1&0 \\ 0&-1&0 \\ 0&0&1 \end{matrix}\middle|\begin{matrix} 0&2 \\ 2&-3 \\ -1&2 \end{matrix}\right) \begin{matrix} +r_2 \\ \cdot(-1) \\ \\ \end{matrix} \sim \left(\begin{matrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{matrix}\middle|\begin{matrix} 2&-1 \\ -2&3 \\ -1&2 \end{matrix}\right)$$

$$\Rightarrow \quad A_{S_1 S_2} = \begin{pmatrix} 2 & -2 & -1 \\ -1 & 3 & 2 \end{pmatrix}$$

2.) Určete $f((2,1,2))$

$$f((2,1,2))_{(S_2)} = \begin{pmatrix} 2 & -2 & -1 \\ -1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad \Rightarrow \quad f((2,1,2)) = (0,5)$$

3.) Určete $\bar{x} \in \mathbb{R}^3$ tak, aby $f(\bar{x}) = (0,3)$

$$f(\bar{x}) = \begin{pmatrix} 2 & -2 & -1 \\ -1 & 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\shortparallel$$

$$\begin{pmatrix} 2x_1 - 2x_2 - x_3 \\ -x_1 + 3x_2 + 2x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

tzn. řešíme soustavu: $\left(\begin{matrix} 2&-2&-1 \\ -1&3&2 \end{matrix}\middle|\begin{matrix} 0 \\ 3 \end{matrix}\right) \begin{matrix} \\ \cdot 2 \end{matrix} \sim \left(\begin{matrix} 2&-2&-1 \\ -2&6&4 \end{matrix}\middle|\begin{matrix} 0 \\ 6 \end{matrix}\right) \begin{matrix} \\ +r_1 \end{matrix} \sim$

$$\left(\begin{matrix} 2&-2&-1 \\ 0&4&3 \end{matrix}\middle|\begin{matrix} 0 \\ 6 \end{matrix}\right)$$

$\Rightarrow \quad 4x_2 + 3x_3 = 6 \quad / x_3 = t \in \mathbb{R}$

$4x_2 + 3t = 6$

$4x_2 = 6 - 3t$

$x_2 = \frac{6}{4} - \frac{3}{4}t$

$\Rightarrow \quad 2x_1 - 2\left(\frac{6}{4} - \frac{3}{4}t\right) - t = 0$

$2x_1 - \frac{6}{2} + \frac{3}{2}t - t = 0$

$2x_1 = \frac{6}{2} - \frac{1}{2}t$

$x_1 = \frac{6}{4} - \frac{1}{4}t$

$$\Rightarrow \bar{x} = \left(\underset{\overset{\shortparallel}{1}}{\frac{6}{4} - \frac{1}{4}t}, \underset{\overset{\shortparallel}{0}}{\frac{6}{4} - \frac{3}{4}t}, \underset{\overset{\shortparallel}{2}}{t}\right), t \in \mathbb{R}$$

4.) Určete jádro lin. zobrazení f, jeho bázi a dimenzi.

$$\mathrm{Ker}\, f = \{ \bar{x} \in \mathbb{R}^3 \mid f(\bar{x}) = (0,0) \}$$

$$\Rightarrow \quad f(\bar{x}) = \begin{pmatrix} 2 & -2 & -1 \\ -1 & 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$ řešíme soustavu

$$\left(\begin{array}{ccc|c} 2 & -2 & -1 & 0 \\ -1 & 3 & 2 & 0 \end{array}\right)\begin{matrix} \\ \cdot 2 \end{matrix} \sim \left(\begin{array}{ccc|c} 2 & -2 & -1 & 0 \\ -2 & 6 & 4 & 0 \end{array}\right)\begin{matrix} \\ +r_1 \end{matrix} \sim \left(\begin{array}{ccc|c} 2 & -2 & -1 & 0 \\ 0 & 4 & 3 & 0 \end{array}\right)$$

$$\Rightarrow \quad 4x_2 + 3x_3 = 0 \qquad / \ \underline{x_3 = t} \qquad\qquad 2x_1 - 2\left(-\tfrac{3}{4}t\right) - t = 0$$

$$4x_2 + 3t = 0 \qquad\qquad 2x_1 + \tfrac{3}{2}t - t = 0$$

$$\underline{x_2 = -\tfrac{3}{4}t} \qquad\qquad 2x_1 + \tfrac{1}{2}t = 0$$

$$\underline{x_1 = -\tfrac{1}{4}t}$$

$$\Rightarrow \mathrm{Ker}\, f = \{ (-\tfrac{1}{4}t, -\tfrac{3}{4}t, t) \mid t \in \mathbb{R} \} = \{ t(-\tfrac{1}{4}, -\tfrac{3}{4}, 1) \mid t \in \mathbb{R} \} = \underline{\underline{\langle (-\tfrac{1}{4}, -\tfrac{3}{4}, 1) \rangle}}$$

$$\text{Báze } \mathrm{Ker}\, f = \{ (-\tfrac{1}{4}, -\tfrac{3}{4}, 1) \}, \quad \dim \underline{\underline{\mathrm{Ker}\, f = 1}}$$

5) Určete obor hodnot lin. zobr. f, jeho bázi a dimenzi

$$H(f) = \{ k_1(0,2) + k_2(1,1) + k_3(-1,2) \mid k_1, k_2, k_3 \in \mathbb{R} \}$$

$$\begin{pmatrix} 0 & 2 \\ 1 & 1 \\ -1 & 2 \end{pmatrix}\begin{matrix} \updownarrow \end{matrix} \sim \begin{pmatrix} -1 & 2 \\ 1 & 1 \\ 0 & 2 \end{pmatrix}\begin{matrix} \\ +r_1 \\ \cdot 3 \end{matrix} \sim \begin{pmatrix} -1 & 2 \\ 0 & 3 \\ 0 & 6 \end{pmatrix}\begin{matrix} \\ \\ -2r_2 \end{matrix} \sim \begin{pmatrix} -1 & 2 \\ 0 & 3 \\ 0 & 0 \end{pmatrix} \Rightarrow$$

$$H(f) = \{ k_1(-1,2) + k_2(0,3) \mid k_1, k_2 \in \mathbb{R} \} = \underline{\underline{\langle (-1,2); (0,3) \rangle}}$$

$$\text{Báze} = \underline{\underline{\{ (-1,2), (0,3) \}}}$$

$$\underline{\underline{\dim H(f) = 2}}$$