

Pr. 11
 Určete matici lineárního zobrazení A vzhledem ke standardním bázím.

1.) $A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $A((x_1, x_2, x_3)) = (2x_1 - x_3, x_2 + 2x_3, x_1 - x_2)$

Stand. báze na \mathbb{R}^3 a \mathbb{R}^3 je $S = \{ \underset{\text{"e}_1}{(1, 0, 0)}; \underset{\text{"e}_2}{(0, 1, 0)}; \underset{\text{"e}_3}{(0, 0, 1)} \}$

$\Rightarrow A(\bar{e}_1) = A((1, 0, 0)) = \begin{pmatrix} 2 \cdot 1 - 0 \\ 0 + 2 \cdot 0 \\ 1 - 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$A(\bar{e}_1)_{\langle S \rangle} = (2, 0, 1)_{\langle S \rangle} = (2, 0, 1)$

$\Rightarrow A(\bar{e}_2) = A((0, 1, 0)) = \begin{pmatrix} 2 \cdot 0 - 0 \\ 1 + 2 \cdot 0 \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$A(\bar{e}_2)_{\langle S \rangle} = (0, 1, -1)_{\langle S \rangle} = (0, 1, -1)$

$\Rightarrow A(\bar{e}_3) = A((0, 0, 1)) = \begin{pmatrix} 2 \cdot 0 - 1 \\ 0 + 2 \cdot 1 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

$A(\bar{e}_3)_{\langle S \rangle} = (-1, 2, 0)_{\langle S \rangle} = (-1, 2, 0)$

\Rightarrow Matice lineárního zobrazení vzhledem ke stand. bázím:

$A_{SS} = \begin{pmatrix} \overset{= A(\bar{e}_1)_{\langle S \rangle}}{2} & \overset{= A(\bar{e}_2)_{\langle S \rangle}}{0} & \overset{= A(\bar{e}_3)_{\langle S \rangle}}{-1} \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$

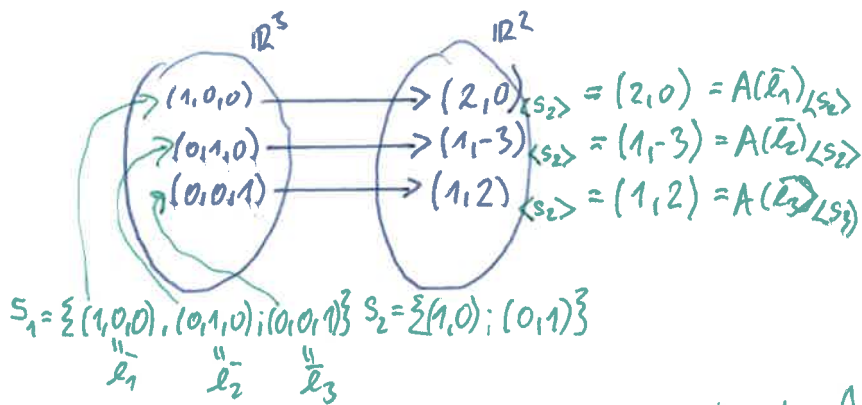
Všimněme si:

$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_3 \\ x_2 + 2x_3 \\ x_1 - x_2 \end{pmatrix} = A(\bar{x})_{\text{ne sloupci}} = A(\bar{x})_{\langle S \rangle}$

$\Rightarrow A(\bar{x})_{\langle S \rangle} = A_{SS} \bar{x}_{\langle S \rangle}$

\Rightarrow např. $A((1, 3, 1)) = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$

$$2.) A: \mathbb{R}^3 \rightarrow \mathbb{R}^2 ; A((x_1, x_2, x_3)) = (2x_1 + x_2 + x_3, -3x_2 + 2x_3)$$

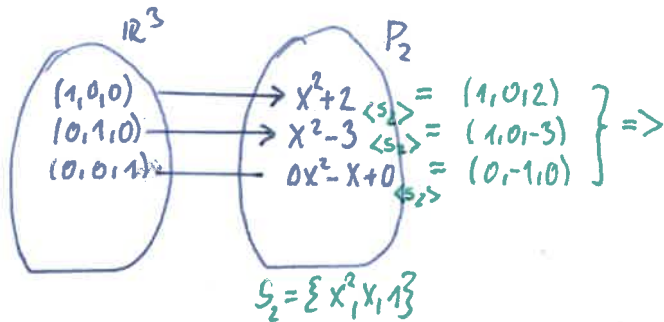


$$\Rightarrow A_{S_1 S_2} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix}$$

$$\text{Např. } A((1,1,1))_{\langle S_2 \rangle} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow A((1,1,1)) = (4, -1)$$

$$3.) A((a,b,c)) = (a+b)x^2 - 2cx + (2a-3b) ; A: \mathbb{R}^3 \rightarrow \mathbb{P}_2$$



$$\Rightarrow A_{S_1 S_2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 2 & -3 & 0 \end{pmatrix}$$

$$\text{Např. } A((1,0,1))_{\langle S_2 \rangle} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 2 & -3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\Rightarrow A((1,0,1)) = x^2 - x + 2$$