Př.: *Určete matici lineárního zobrazení A vzhledem ke standardním bázím.*

1.) $A: \mathbb{R}^3 \to \mathbb{R}^3$, $\quad A((x_1, x_2, x_3)) = (2x_1 - x_3,\ x_2 + 2x_3,\ x_1 - x_2)$

Stand. báze na $\mathbb{R}^3$ a $\mathbb{R}^3$ je $S = \{\underbrace{(1,0,0)}_{=\bar{e}_1}; \underbrace{(0,1,0)}_{=\bar{e}_2}; \underbrace{(0,0,1)}_{=\bar{e}_3}\}$

$$\Rightarrow A(\bar{e}_1) = A((\underset{x_1}{1}, \underset{x_2}{0}, \underset{x_3}{0})) = (\underbrace{2\cdot 1 - 0}_{2x_1 - x_3}, \underbrace{0 + 2\cdot 0}_{x_2 + 2x_3}, \underbrace{1 - 0}_{x_1 - x_2}) = (2, 0, 1)$$

$$A(\bar{e}_1)_{\langle S \rangle} = (2, 0, 1)_{\langle S \rangle} = (2, 0, 1)$$

$$\Rightarrow A(\bar{e}_2) = A((\underset{x_1}{0}, \underset{x_2}{1}, \underset{x_3}{0})) = (2\cdot 0 - 0,\ 1 + 2\cdot 0,\ 0 - 1) = (0, 1, -1)$$

$$A(\bar{e}_2)_{\langle S \rangle} = (0, 1, -1)_{\langle S \rangle} = (0, 1, -1)$$

$$\Rightarrow A(\bar{e}_3) = A((0,0,1)) = (2\cdot 0 - 1,\ 0 + 2\cdot 1,\ 0 - 0) = (-1, 2, 0)$$

$$A(\bar{e}_3)_{\langle S \rangle} = (-1, 2, 0)_{\langle S \rangle} = (-1, 2, 0)$$

$\Rightarrow$ *Matice lineárního zobrazení vzhledem ke stand. bázím:*

$$A_{SS} = \begin{array}{ccc} = A(\bar{e}_1)_{\langle S \rangle} & = A(\bar{e}_2)_{\langle S \rangle} & = A(\bar{e}_3)_{\langle S \rangle} \\ \end{array}$$
$$A_{SS} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

Všimněme si:

$$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\bar{x}_{\langle S \rangle}} = \begin{pmatrix} 2x_1 - x_3 \\ x_2 + 2x_3 \\ x_1 - x_2 \end{pmatrix} = A(\bar{x}) \text{ ve sloupci} = A(\bar{x})_{\langle S \rangle}$$

$$\Rightarrow A(\bar{x})_{\langle S \rangle} = A_{SS}\, \bar{x}_{\langle S \rangle}$$

$\Rightarrow$ např. $A((1,3,1)) = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$

2.) $A: \mathbb{R}^3 \to \mathbb{R}^2 \;;\; A((x_1,x_2,x_3)) = (2x_1+x_2+x_3,\ -3x_2+2x_3)$

$\mathbb{R}^3 \qquad\qquad \mathbb{R}^2$

$(1,0,0) \to (2,0)_{\langle S_2\rangle} = (2,0) = A(\bar{e}_1)_{\langle S_2\rangle}$

$(0,1,0) \to (1,-3)_{\langle S_2\rangle} = (1,-3) = A(\bar{e}_2)_{\langle S_2\rangle}$

$(0,0,1) \to (1,2)_{\langle S_2\rangle} = (1,2) = A(\bar{e}_3)_{\langle S_2\rangle}$

$\Rightarrow A_{S_1S_2} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix}$

$S_1 = \{\underset{\bar{e}_1}{(1,0,0)}, \underset{\bar{e}_2}{(0,1,0)}; \underset{\bar{e}_3}{(0,0,1)}\}$ $\quad S_2 = \{(1,0);(0,1)\}$

Např. $A((1,1,1))_{\langle S_2\rangle} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \Rightarrow$

$\Rightarrow A((1,1,1)) = (4,-1)$

3.) $A((a,b,c)) = (a+b)x^2 - 2cx + (2a-3b) \;;\; A: \mathbb{R}^3 \to P_2$

$\mathbb{R}^3 \qquad\qquad P_2$

$(1,0,0) \to x^2+2_{\langle S_2\rangle} = (1,0,2)$

$(0,1,0) \to x^2-3_{\langle S_2\rangle} = (1,0,-3)$

$(0,0,1) \to 0x^2-x+0_{\langle S_2\rangle} = (0,-1,0)$

$\Rightarrow \quad A_{S_1S_2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 2 & -3 & 0 \end{pmatrix}$

$S_2 = \{x^2, x, 1\}$

Např. $A((1,0,1))_{\langle S_2\rangle} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 2 & -3 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$\Rightarrow A((1,0,1)) = x^2 - x + 2$