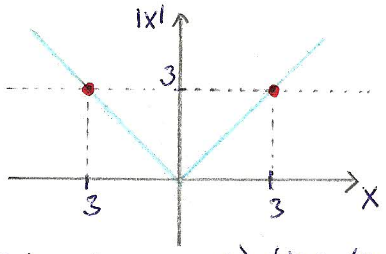
Př: Určete všechna $x \in \mathbb{R}$ splňující dané rovnice a nerovnice.

1) $|x| = 3$



a) $\underline{|x \in (-\infty, 0)|}$ ⇒ $|x| = -x$ (př: $|-3| = -(-3) = 3$)

$|x| = 3$

$-x = 3$

$\underline{\underline{x = -3}}$

b) $\underline{|x \in \langle 0, \infty)|}$ ⇒ $|x| = x$ (př: $|5| = 5$)

$|x| = 3$

$\underline{\underline{x = 3}}$

⇒ $\underline{\underline{x \in \{-3, 3\}}}$

2) $|x| > 3$

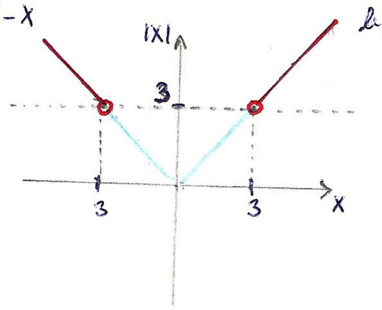
a) $\underline{|x \in (-\infty, 0)|}$ ⇒ $|x| = -x$

$|x| > 3$

$-x > 3 \quad (-1)$

$x < -3$

$\underline{x \in (-\infty, -3)}$



b) $\underline{|x \in \langle 0, \infty)|}$ ⇒ $|x| = x$

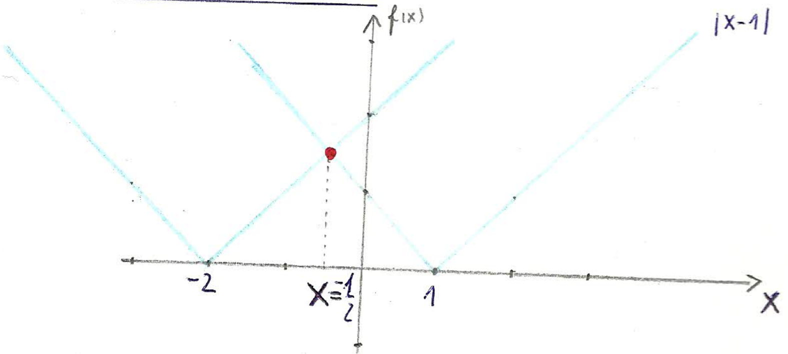
$|x| > 3$

$x > 3$

$\underline{x \in (3, \infty)}$

$\underline{\underline{x \in (3, \infty) \cup (-\infty, -3)}}$

3.) $|x-1| = |2+x|$



nulové body:

a) $\underline{|x \in (-\infty, -2)|}$ ⇒ $|x-1| = -(x-1)$; $|2+x| = -(2+x)$

$-(x-1) = -(2+x)$

$-x+1 = -x-2$

$1 = -2$

⇓

$\underline{\text{v intervalu } (-\infty,-2) \text{ rovnice nemá řešení}}$

b) $\underline{|x \in \langle -2, 1)|}$ ⇒ $|x-1| = -(x-1)$

$|2+x| = 2+x$

$-(x-1) = 2+x$

$-x+1 = 2+x$

$-2x = 1$

$\underline{x = -\frac{1}{2}}$

c) $\underline{|x \in \langle 1, \infty)|}$ ⇒ $|x-1| = x-1$

$|2+x| = 2+x$

$x-1 = 2+x$

$-1 = 2$

$\underline{\text{v intervalu } \langle 1,\infty) \text{ rovnice nemá řešení}}$

⇒ $\underline{\underline{x = -\frac{1}{2}}}$

Př: Určete všechna $x \in \mathbb{R}$ splňující dané rovnice a nerovnice

1.) $2(x-1) + |2x-1| = 3 + |3x-6|$

$2x-1=0$ $\quad$ $3x-6=0$

$2x=1$ $\quad$ $3x=6$

$\underline{x=\frac{1}{2}}$ $\quad$ $\underline{x=2}$

Nulové body:

a) $\boxed{x \in (-\infty, \frac{1}{2})} \Rightarrow |2x-1| = -(2x-1)$

$|3x-6| = -(3x-6)$ (dosadíme např. $x=0$ $\Rightarrow$ $2x-1=-1<0$, $3x-6=-6<0$)

$2(x-1) - (2x-1) = 3 - (3x-6)$

$2x-2 \; -2x+1 = 3-3x+6$

$-1 = -3x+9$

$3x = 10$

$x = \frac{10}{3} = 3+\frac{1}{3} \notin (-\infty, \frac{1}{2})$ $\Rightarrow$ rovnice nemá v intervalu $(-\infty, \frac{1}{2})$ řešení!

b) $\boxed{x \in \langle \frac{1}{2}, 2)} \Rightarrow |2x-1| = (2x-1)$

$|3x-6| = -(3x-6)$

$2(x-1) + (2x-1) = 3 - (3x-6)$

$2x-2+2x-1 = 3-3x+6$

$4x-3 = 9-3x$

$7x = 12$

$\underline{x = \frac{12}{7}} \in \langle \frac{1}{2}, 2)$ (je řešením rovnice)

c) $\boxed{x \in \langle 2, \infty)} \Rightarrow |2x-1| = 2x-1$

$|3x-6| = 3x-6$

$2(x-1) + (2x-1) = 3+3x-6$

$2x-2+2x-1 = 3x-3$

$4x-3 = 3x-3$

$x = 0 \notin \langle 2, \infty)$ $\Rightarrow$ rovnice nemá v intervalu $\langle 2, \infty)$ řešení

$\Rightarrow$ $\underline{\underline{x = \frac{12}{7}}}$ (Je jediné řešení zadané rovnice)

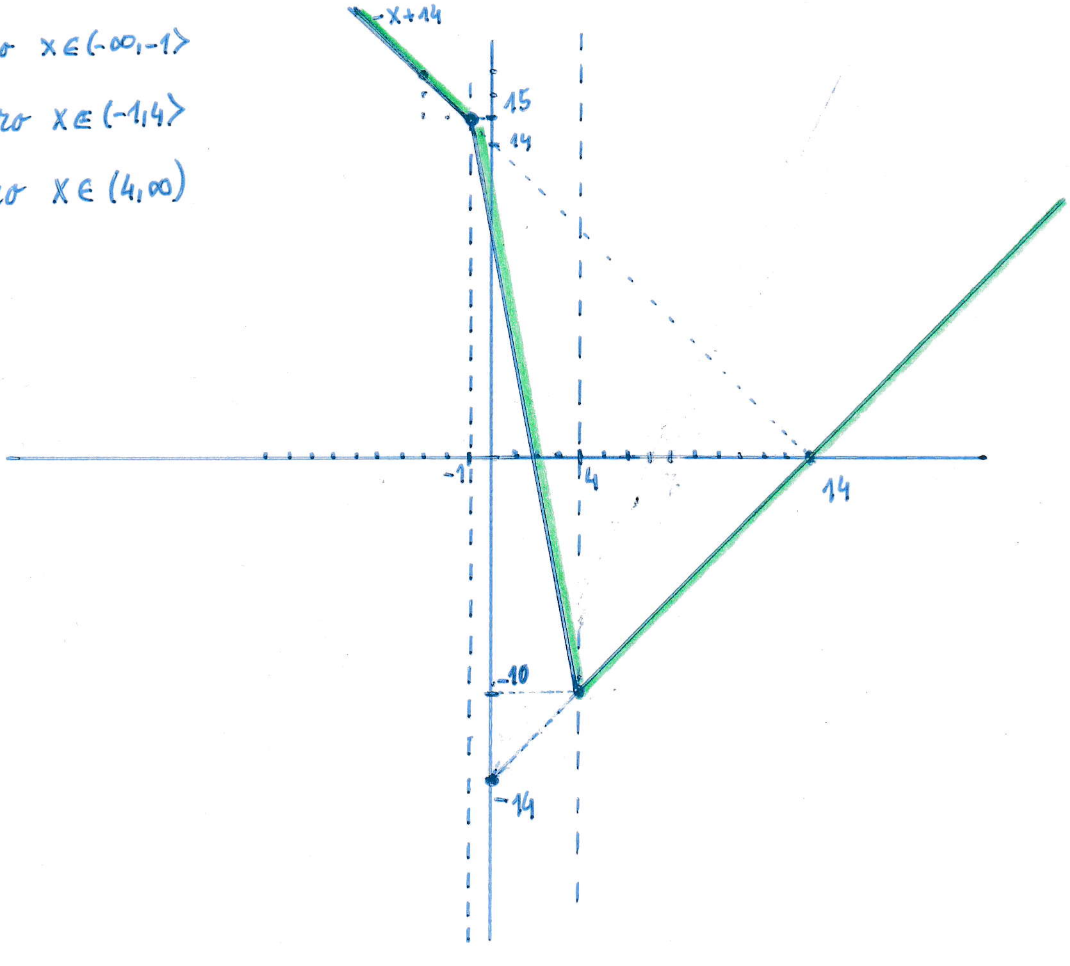
Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $3|x-4| - 2|x+1| \geq 0$.

$x \in (-\infty, -1\rangle$	-1	$x \in (-1, 4\rangle$	4	$x \in (4, \infty)$
$\lvert x-4\rvert = -(x-4)$ $\lvert x+1\rvert = -(x+1)$		$\lvert x-4\rvert = -(x-4)$ $\lvert x+1\rvert = x+1$		$\lvert x-4\rvert = x-4$ $\lvert x+1\rvert = x+1$
$-3(x-4) + 2(x+1) \geq 0$ $-3x + 12 + 2x + 2 \geq 0$ $-x + 14 \geq 0$ $x \leq 14$		$-3(x-4) - 2(x+1) \geq 0$ $-3x + 12 - 2x - 2 \geq 0$ $-5x + 10 \geq 0$ $5x \leq 10$ $x \leq 2$		$3(x-4) - 2(x+1) \geq 0$ $3x - 12 - 2x - 2 \geq 0$ $x - 14 \geq 0$ $x \geq 14$
$\Rightarrow \quad x \in (-\infty, -1\rangle$		$x \in (-1, 2\rangle$		$x \in \langle 14, \infty)$

$$\Rightarrow x \in (-\infty, -1\rangle \cup (-1, 2\rangle \cup \langle 14, \infty) = \underline{\underline{(-\infty, 2\rangle \cup \langle 14, \infty)}}$$

Př. Načrtněte graf funkce $f(x) = 3|x-4| - 2|x+1|$

$$f(x) = \begin{cases} -x + 14 & \text{pro } x \in (-\infty, -1\rangle \\ -5x + 10 & \text{pro } x \in (-1, 4\rangle \\ x - 14 & \text{pro } x \in (4, \infty) \end{cases}$$



Př: Nalezněte všechna $x \in \mathbb{R}$ splňující $\frac{|x^2-x|}{|x-1|} = x$.

Řešíme pro $x \neq 1$.

$$\frac{|x^2-x|}{|x-1|} = x$$

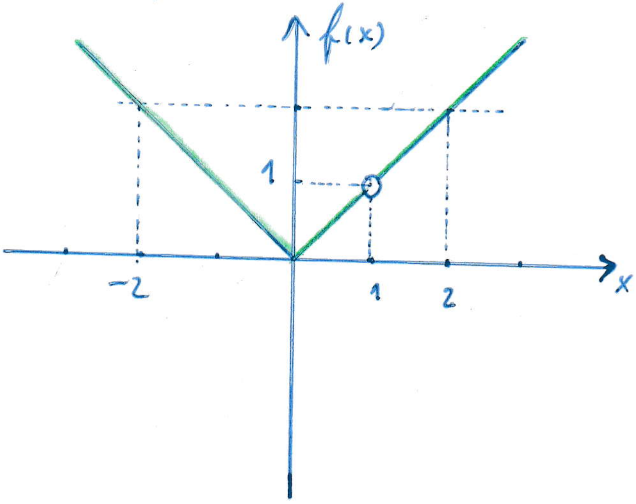
$$\frac{|x\cdot(x-1)|}{|x-1|} = x \quad /(\text{pro } x \neq 1)$$

$$|x| = x$$

$$\Rightarrow x \in \langle 0,\infty) - \{1\} = \underline{\underline{\langle 0,1) \cup (1,\infty)}}$$

Př: Načrtněte graf funkce $f(x) = \frac{|x^2-x|}{|x-1|}$

$f(x) = |x|$ pro $x \in \mathbb{R} - \{1\}$, $1 \notin D(f)$



Př. Nalezněte všechna $x\in\mathbb{R}$ splňující $|1+x|\cdot|1-x| = x\cdot|x|$

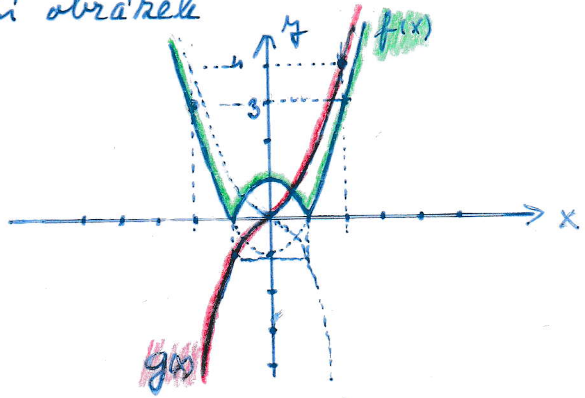
$1-x \geq 0 \Leftrightarrow 1 > x$

$x\in(-\infty,-1\rangle$ -1	$x\in(-1,0\rangle$ 0	$x\in(0,1\rangle$ 1	$x\in(1,\infty)$
$\|1+x\| = -(1+x)$ $\|1-x\| = 1-x$ $\|x\| = -x$	$\|1+x\| = 1+x$ $\|1-x\| = 1-x$ $\|x\| = -x$	$\|1+x\| = 1+x$ $\|1-x\| = 1-x$ $\|x\| = x$	$\|1+x\| = 1+x$ $\|1-x\| = -(1-x)$ $\|x\| = x$
$-(1+x)(1-x) = -x^2$ $-(1-x^2) = -x^2$ $-1+x^2 = -x^2$ $1 = 2x^2$ $x = \pm\frac{1}{\sqrt{2}} \doteq \pm 0{,}707$ $\notin(-\infty,-1\rangle$ $\emptyset$	$(1+x)(1-x) = -x^2$ $1-x^2 = -x^2$ $1 = 0$ $\emptyset$	$(1+x)(1-x) = x^2$ $1-x^2 = x^2$ $1 = 2x^2$ $x^2 = \frac{1}{2}$ $x = -\frac{1}{\sqrt{2}} \notin (0,1\rangle$ $x = \underline{\underline{\frac{1}{\sqrt{2}}}} \in (0,1\rangle$	$-(1+x)(1-x) = x^2$ $-(1-x^2) = x^2$ $-1+x^2 = x^2$ $-1 = 0$ $\emptyset$

$$\Rightarrow \underline{\underline{x = \frac{1}{\sqrt{2}}}}$$

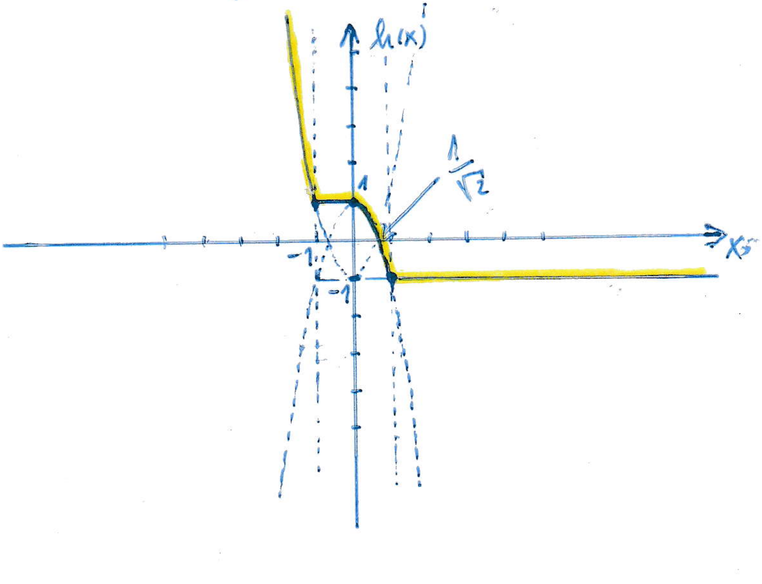
Př. Do jednoho obrázku namalujte grafy funkcí $f(x) = |1+x||1-x|$ a $g(x) = x\cdot|x|$ a $f(x)-g(x) = h(x)$ na další obrázek

$$f(x) = |1-x^2| = \begin{cases} 1-x^2 & \text{pro } x\in\langle -1,1\rangle \\ -1+x^2 & \text{pro } x\in(-\infty,-1)\cup(1,\infty) \end{cases}$$



$$g(x) = \begin{cases} x^2 & \text{pro } x \geq 0 \\ -x^2 & \text{pro } x < 0 \end{cases}$$

$$h(x) = \begin{cases} -1+2x^2 & \text{pro } x\in(-\infty,-1\rangle \\ 1 & \text{pro } x\in(-1,0\rangle \\ 1-2x^2 & \text{pro } x\in(0,1\rangle \\ -1 & \text{pro } x\in(1,\infty) \end{cases}$$



Př: Nalezněte všechna $x \in \mathbb{R}$ splňující: $x^2 + 3|x| + 2 = 0$.

a) $x \in \langle 0, \infty)$ $\Rightarrow$ $|x| = x$ $\Rightarrow$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0 \Rightarrow x = \begin{cases} -2 \notin \langle 0,\infty) \\ -1 \notin \langle 0,\infty) \end{cases} \Bigg\} \Rightarrow$ v intervalu $\langle 0, \infty)$ jsme řešení nenašli

b) $x \in (-\infty, 0)$ $\Rightarrow$ $|x| = -x$

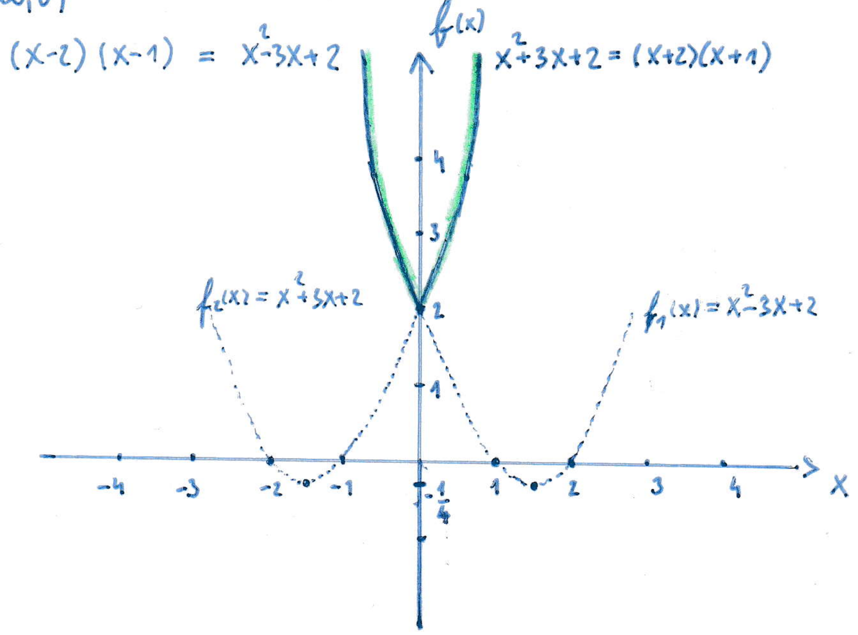
$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0 \Rightarrow x = \begin{cases} 2 \notin (-\infty,0) \\ 1 \in (-\infty,0) \end{cases} \Bigg\} \Rightarrow$ v intervalu $(-\infty, 0)$ jsme řešení také nenašli

$\Rightarrow$ Zadaná rovnice nemá řešení (v $\mathbb{R}$).

Př: načrtněte graf funkce $f(x) = x^2 + 3|x| + 2$

$$f(x) = \begin{cases} x^2 + 3x + 2 & \text{pro } x \in \langle 0, \infty) \\ x^2 - 3x + 2 & \text{pro } x \in (-\infty, 0) \end{cases}$$



$f(0) = 2$

$f_1(\frac{3}{2}) = (\frac{3}{2})^2 - 3\frac{3}{2} + 2 = \frac{9-18+8}{4} = -\frac{1}{4}$

$f_2(-\frac{3}{2}) = (-\frac{3}{2})^2 + 3(-\frac{3}{2}) + 2 = \frac{9-18+8}{4} = -\frac{1}{4}$

(je to sudá funkce)

Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $-x^2 + 2|x+2| - 4 = 0$

a) $x \in \langle -2, \infty) \Rightarrow |x+2| = x+2$

$$-x^2 + 2(x+2) - 4 = 0$$
$$-x^2 + 2x + 4 - 4 = 0$$
$$-x^2 + 2x = 0$$
$$-x(x-2) = 0 \Rightarrow x = \begin{cases} \underline{\underline{0}} \in \langle -2, \infty) \\ \underline{\underline{2}} \in \langle -2, \infty) \end{cases}$$

b) $x \in (-\infty, -2) \Rightarrow |x+2| = -(x+2)$

$$-x^2 - 2(x+2) - 4 = 0$$
$$-x^2 - 2x - 4 - 4 = 0$$
$$-x^2 - 2x - 8 = 0 \quad / \cdot (-1)$$
$$x^2 + 2x + 8 = 0$$
$$x_{1,2} = \frac{-2 \pm \sqrt{\overset{=D}{\boxed{4 - 4 \cdot 8}}}}{2} \Rightarrow D < 0 \Rightarrow \text{žádné řešení v } \mathbb{R}!$$

$\Rightarrow$ Všechna řešení rovnice jsou $x = 0$ a $x = 2$.

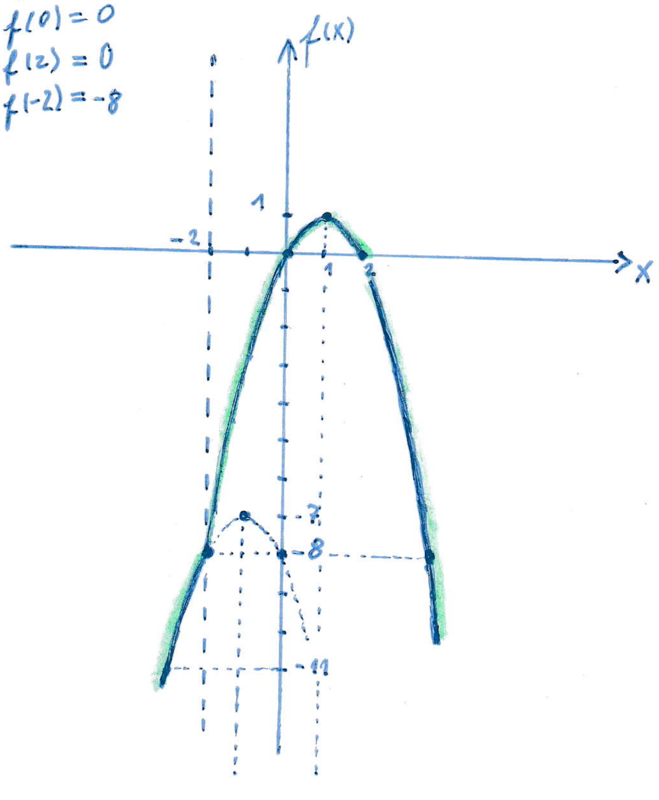
Př. Načrtněte graf funkce $f(x) = -x^2 + 2|x+2| - 4$

a) pro $x \in \langle -2, \infty)$ je $f(x) = -x^2 + 2(x+2) - 4 =$
$= -x^2 + 2x = -x(x-2)$

$\Big\}$ $f(1) = 1$
$f(0) = 0$
$f(2) = 0$
$f(-2) = -8$

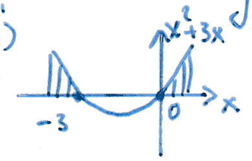
b) pro $x \in (-\infty, -2)$ je $f(x) = -x^2 - 2(x+2) - 4 =$
$= -x^2 - 2x - 8 =$
$= -(x^2 + 2x + 8) =$
$= -((x+1)^2 + 7)$

$\Rightarrow f(-3) = -((-3+1)^2 + 7) = -11$



Př. Naleznete všechna řešení ($x\in\mathbb{R}$) rovnice $|x^2+3x|+x-5=0$

a) $\underset{\underset{x\cdot(x+3)}{\shortparallel}}{x^2+3x} \geq 0$ je splněno pro $x \in (-\infty,-3\rangle \cup \langle 0,\infty)$



$\Rightarrow$ Pro tato x je $|x^2+3x| = (x^2+3x)$ $\Rightarrow$ Řešíme rovnici:

$x^2+3x+x-5=0$

$x^2+4x-5=0$

$(x+5)(x-1)=0 \Rightarrow x =$

$\underline{\underline{-5}} \in (-\infty,-3\rangle\cup\langle 0,\infty) \Rightarrow$ je to řešení

$\underline{\underline{1}} \in (-\infty,-3\rangle\cup\langle 0,\infty) \Rightarrow$ je to řešení

b) $x^2+3x<0$ je splněno pro $x\in(-3,0)$ $\Rightarrow$ pro tato x řešíme:

$-(x^2+3x)+x-5=0$

$-x^2-2x-5=0$

$-(x^2+2x+5)=0$

$-((x+1)^2+4)=0 \Rightarrow$ v $\mathbb{R}$ nemá řešení

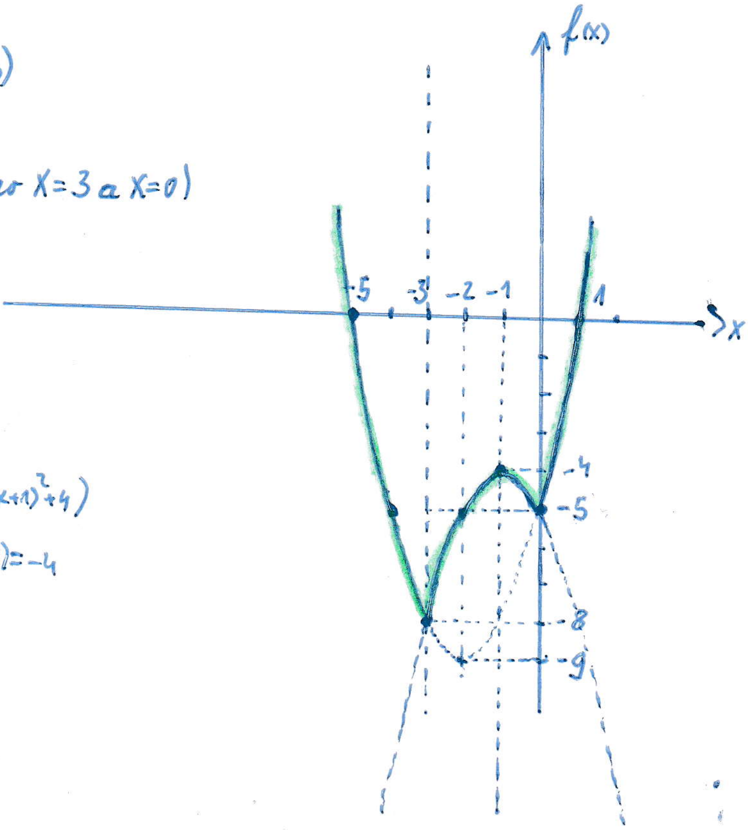
$\Rightarrow$ **Zadaná rovnice je splněna jen pro $x=-5$ a $x=1$.**

Př. Načrtněte graf funkce $f(x)=|x^2+3x|+x-5$

$$f(x)=\begin{cases} x^2+4x-5=(x+5)(x-1) & \text{pro } x\in(-\infty,-3\rangle\cup\langle 0,\infty) \\ -((x+1)^2+4) & \text{pro } x\in(-3,0) \end{cases}$$

(ale i pro $x=3$ a $x=0$)

$f(1)=0$
$f(-5)=0$
$f(0)=|0+0|+0-5=-5$

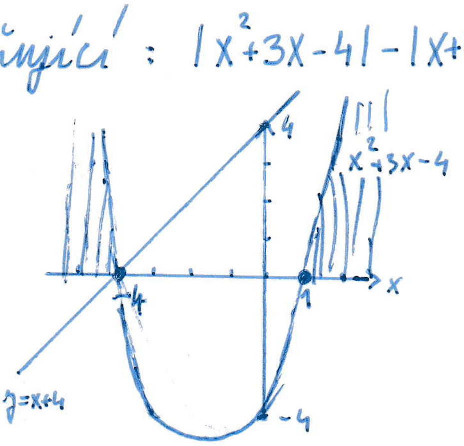


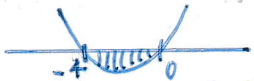
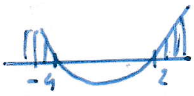
$f_1(x)=\underset{\underset{(x+5)(x-1)}{\shortparallel}}{x^2+4x-5}$ $\Rightarrow$ $f_1(-2)=4-8-5=-9$
$f_1(0)=-5$
$f_1(-3)=9-12-5=-8$
$f_1(-4)=-5$

$f_2(x)=-((x+1)^2+4)$
$f_2(-1)=-4$

Př. Nalezněte všechna $x \in \mathbb{R}$ splňující: $|x^2+3x-4|-|x+4| \geq 0$.

$x^2+3x-4 = (x+4)(x-1)$ $\Rightarrow$

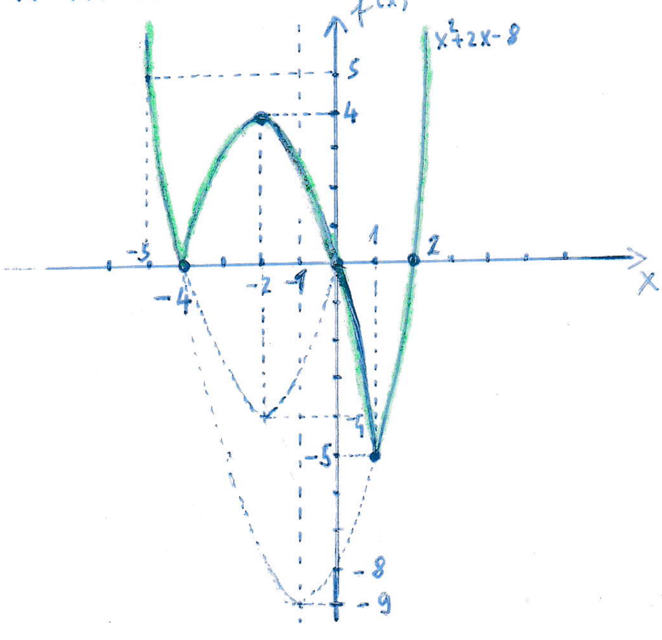


$x \in (-\infty,-4\rangle$	$x \in (-4,1\rangle$	$x \in (1,\infty)$
$\lvert x^2+3x-4 \rvert = x^2+3x-4$ $\lvert x+4 \rvert = -x-4$	$\lvert x^2+3x-4 \rvert = -x^2-3x+4$ $\lvert x+4 \rvert = x+4$	$\lvert x^2+3x-4 \rvert = x^2+3x-4$ $\lvert x+4 \rvert = x+4$
$x^2+3x-4-(-x-4) \geq 0$ $x^2+4x \geq 0$ $x(x+4) \geq 0$  $x \in [(-\infty,-4\rangle \cup \langle 0,\infty)] \cap (-\infty,-4\rangle$ $x \in (-\infty,-4\rangle$	$-x^2-3x+4-(x+4) \geq 0$ $-x^2-4x \geq 0 \quad \lvert \cdot(-1)$ $x^2+4x \leq 0$ $x(x+4) \leq 0$  $x \in \langle -4,0\rangle \cap (-4,1\rangle$ $x \in \langle -4,0\rangle$	$x^2+3x-4-(x+4) \geq 0$ $x^2+2x-8 \geq 0$ $(x+4)(x-2) \geq 0$  $x \in [(-\infty,-4\rangle \cup \langle 2,\infty)] \cap (1,\infty)$ $x \in \langle 2,\infty)$

$\Rightarrow$ Nerovnice je splněna pro $x \in \overbrace{(-\infty,-4\rangle \cup \langle -4,0\rangle}^{(-\infty,0\rangle} \cup \langle 2,\infty)$

Př. Načrtněte graf funkce $f(x) = |x^2+3x-4|-|x+4|$

$$f(x) = \begin{cases} x(x+4) & \text{pro } x \in (-\infty,-4\rangle \\ -x^2-4x = -x(x+4) & \text{pro } x \in (-4,1\rangle \\ x^2+2x-8 = (x+4)(x-2) & \text{pro } x \in (1,\infty) \end{cases}$$



Př: Nalezněte všechna $x \in \mathbb{R}$ splňující $-x^2 + 3|x-4| - 8 \geq 0$

$x \in (-\infty, 4\rangle$	$x \in (4, \infty)$
$\lvert x-4\rvert = -(x-4)$	$\lvert x-4\rvert = x-4$

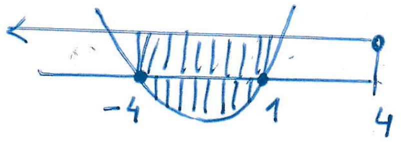
$x \in (-\infty, 4\rangle$:

$$-x^2 - 3(x-4) - 8 \geq 0$$

$$-x^2 - 3x + 4 \geq 0 \quad / \cdot (-1)$$

$$x^2 + 3x - 4 \leq 0$$

$$(x+4)(x-1) \leq 0$$



$x \in \langle -4, 1\rangle$

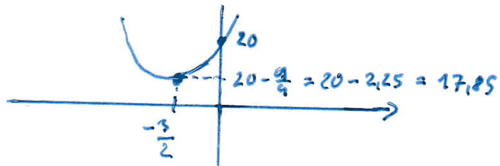
$x \in (4, \infty)$:

$$-x^2 + 3(x-4) - 8 \geq 0$$

$$-x^2 + 3x - 20 \geq 0$$

$$x^2 - 3x + 20 \leq 0$$

$$(x - \tfrac{3}{2})^2 - \tfrac{9}{4} + 20 \leq 0$$



$x \in \emptyset$

$$\Rightarrow \quad \underline{\underline{x \in \langle -4, 1\rangle}}$$

Př: Načrtněte graf funkce $f(x) = -x^2 + 3|x-4| - 8$

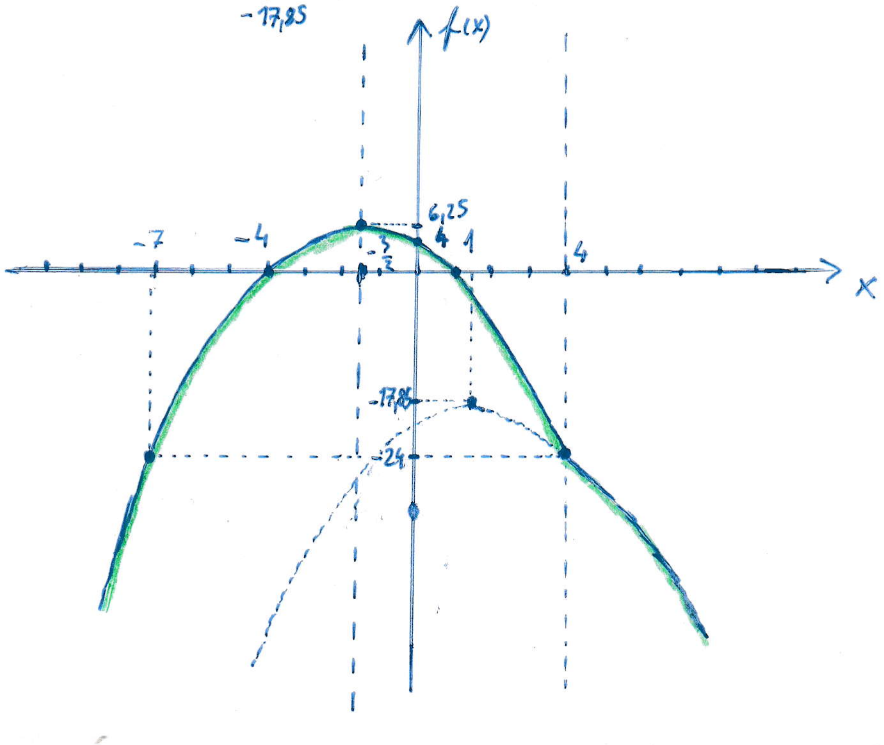
$$f(x) = \begin{cases} -x^2 - 3(x-4) - 8 = -x^2 - 3x + 4 = -(x+4)(x-1) & \text{pro } x \in (-\infty, 4\rangle \\ -x^2 + 3(x-4) - 8 = -x^2 + 3x - 20 = -(x - \frac{3}{2})^2 + \underbrace{\frac{9}{4} - 20}_{-17{,}85} & \text{pro } x \in (4, \infty) \end{cases}$$

$f(4) = -16 - 8 = -24$

$f(0) = -8$

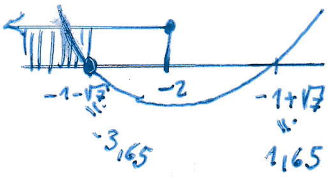
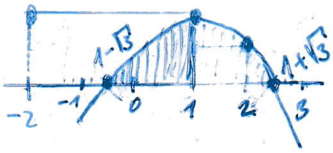
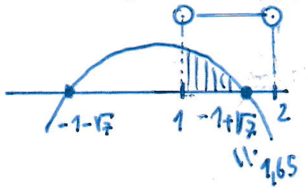
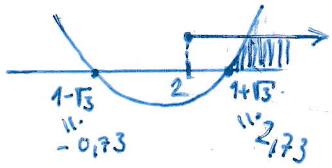
$f(-\frac{3}{2}) = -\frac{9}{4} + 3\,\frac{3}{2} + 4 =$

$= \frac{-9 + 18 + 16}{4} = \frac{25}{4} = 6{,}25$



Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $|x^2-4|-2|x-1| \geq 0$.

$$|x^2-4| \begin{cases} (x^2-4) & \text{pro } x \in (-\infty,-2\rangle \cup \langle 2,\infty) \\ -(x^2-4) & \text{pro } x \in (-2,2) \end{cases} \qquad |x-1| \begin{cases} (x-1) & \text{pro } x \in (1,\infty) \\ -(x-1) & \text{pro } x \in (-\infty,1\rangle \end{cases}$$

$x \in (-\infty,-2\rangle$ -2	$x \in (-2,1\rangle$ 1	$x \in (1,2)$ 2	$x \in \langle 2,\infty)$
$\lvert x^2-4\rvert = x^2-4$ $\lvert x-1\rvert = -(x-1)$	$\lvert x^2-4\rvert = -(x^2-4)$ $\lvert x-1\rvert = -(x-1)$	$\lvert x^2-4\rvert = -(x^2-4)$ $\lvert x-1\rvert = x-1$	$\lvert x^2-4\rvert = x^2-4$ $\lvert x-1\rvert = x-1$
$(x^2-4)+2(x-1) \geq 0$ $x^2+2x-6 \geq 0$ $x_{1,2} = \frac{-2 \pm \sqrt{4+24}}{2} = \begin{cases} -1-\sqrt{7} \\ -1+\sqrt{7} \end{cases}$	$-(x^2-4)+2(x-1) \geq 0$ $-x^2+2x+2 \geq 0$ $x_{1,2} = \frac{-2 \pm \sqrt{4+8}}{-2} = \begin{cases} 1-\sqrt{3} \\ 1+\sqrt{3} \end{cases}$	$-(x^2-4)-2(x-1)$ $-x^2-2x+6 \geq 0$ $x_{1,2} = \frac{2 \pm \sqrt{4+24}}{-2} = \begin{cases} -1-\sqrt{7} \\ -1+\sqrt{7} \end{cases}$	$(x^2-4)-2(x-1) \geq 0$ $x^2-2x-2 \geq 0$ $x_{1,2} = \frac{2 \pm \sqrt{4+8}}{2} = \begin{cases} 1-\sqrt{3} \\ 1+\sqrt{3} \end{cases}$
			
$x \in (-\infty,-1-\sqrt{7}\rangle$	$x \in \langle 1-\sqrt{3},1\rangle$	$x \in (1,-1+\sqrt{7}\rangle$	$x \in \langle 1+\sqrt{3},\infty)$

$$\Rightarrow \underline{\underline{x \in (-\infty,-1-\sqrt{7}\rangle \cup \langle 1-\sqrt{3},-1+\sqrt{7}\rangle \cup \langle 1+\sqrt{3},\infty)}}$$

Př: Načrtněte graf funkce $f(x) = |x^2-4|-2|x-1|$

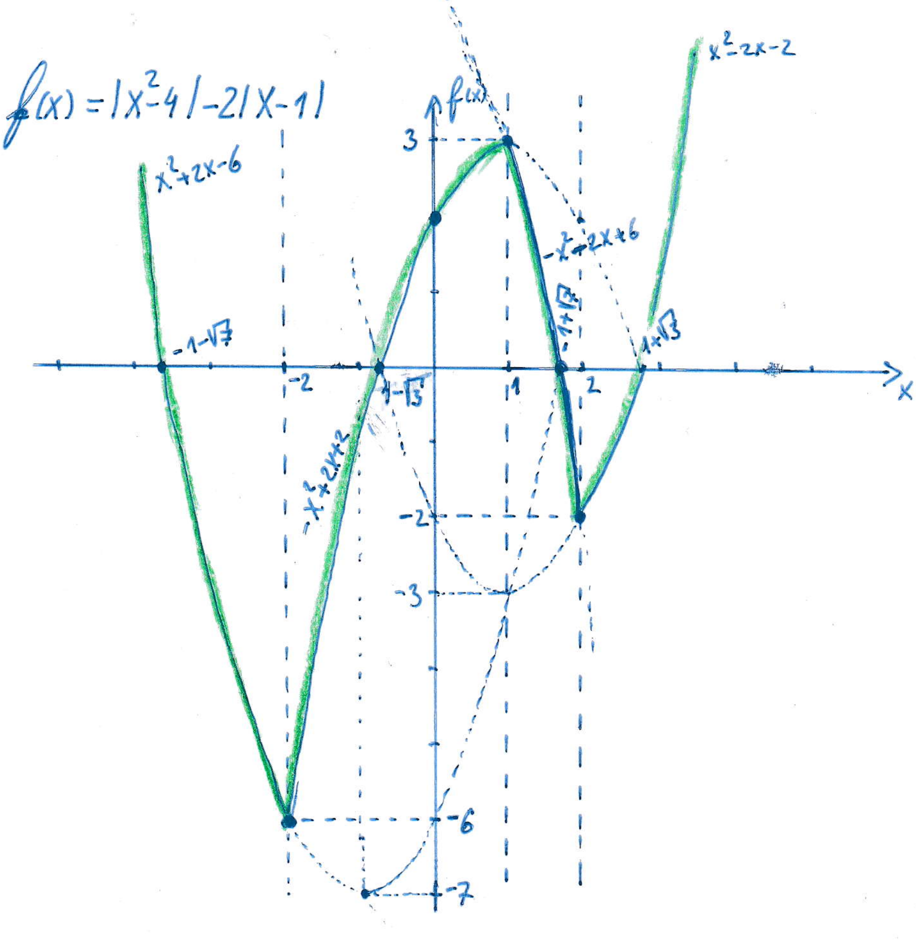
$$f(x) = \begin{cases} x^2+2x-6 & \text{pro } x \in (-\infty,-2\rangle \\ -x^2+2x+2 & \text{pro } x \in (-2,1\rangle \\ -x^2-2x+6 & \text{pro } x \in (1,2) \\ x^2-2x-2 & \text{pro } x \in \langle 2,\infty) \end{cases}$$

$f(-2) = -6$

$f(1) = 3$

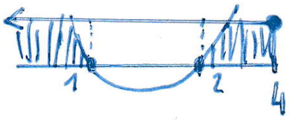
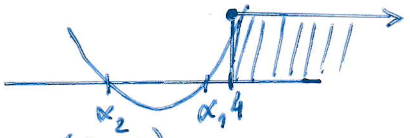
$f(2) = -2$

$f(0) = 2$



Př. Nalezněte všechna $x \in \mathbb{R}$ splňující $x^2 + 3|x-4| - 10 \geq 0$

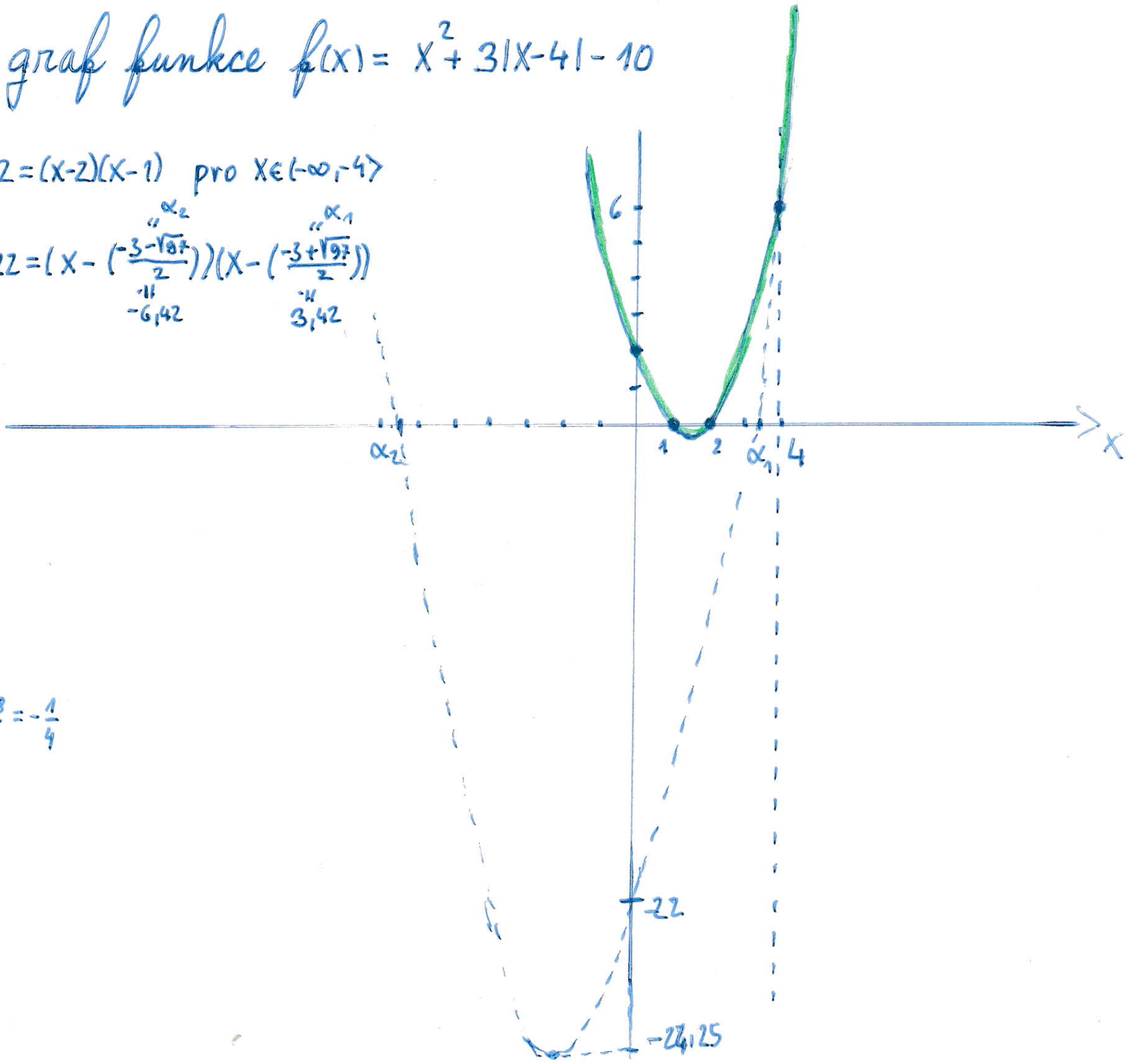
4

$x \in (-\infty, 4>$	$x \in <4, \infty)$
$\lvert x-4 \rvert = -(x-4)$	$\lvert x-4 \rvert = (x-4)$
$x^2 - 3(x-4) - 10 \geq 0$ $x^2 - 3x + 2 \geq 0$ $(x-2)(x-1) \geq 0$  $x \in (-\infty, 1> \cup <2, 4>$	$x^2 + 3(x-4) - 10 \geq 0$ $x^2 + 3x - 22 \geq 0$ $x_{1,2} = \frac{-3 \pm \sqrt{9+88}}{2} = \begin{cases} \frac{-3+\sqrt{97}}{2} \doteq 3{,}42 = \alpha_1 \\ \frac{-3-\sqrt{97}}{2} \doteq -6{,}42 = \alpha_2 \end{cases}$  $x \in <4, \infty)$

$\Rightarrow$ $\underline{\underline{x \in (-\infty, 1> \cup <2, \infty)}}$

Př. Načrtněte graf funkce $f(x) = x^2 + 3|x-4| - 10$

$$f(x) = \begin{cases} x^2 - 3x + 2 = (x-2)(x-1) \quad \text{pro } x \in (-\infty, -4> \\ x^2 + 3x - 22 = \Big(x - \big(\underset{\substack{\shortparallel \\ -6{,}42}}{\overset{\text{„}\alpha_2\text{“}}{\tfrac{-3-\sqrt{97}}{2}}}\big)\Big)\Big(x - \big(\underset{\substack{\shortparallel \\ 3{,}42}}{\overset{\text{„}\alpha_1\text{“}}{\tfrac{-3+\sqrt{97}}{2}}}\big)\Big) \end{cases}$$



$x^2 + 3x - 22 = -24{,}25$

$f(\frac{3}{2}) = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9 - 18 + 8}{4} = -\frac{1}{4}$

2) $3|2x-6|+4|x| \geq x-|x+1|$

$2x-6=0 \iff x=3$
$x=0 \iff x=0$
$x+1=0 \iff x=-1$

Nulové body:

a) $\underline{|x \in (-\infty,-1)|}$

$|2x-6| = -(2x-6)$
$|x| = -x$
$|x+1| = -(x+1)$

$-3(2x-6)-4x \geq x+(x+1)$

$-6x+18-4x \geq 2x+1$

$-12x \geq -17$

$x \leq \frac{17}{12}$ $\Rightarrow$ to splňují $\underline{\text{všechna } x \in (-\infty,-1)}$

$\overset{\shortparallel}{1+\frac{5}{12}}$

b) $\underline{|x \in \langle -1,0)|}$

$|2x-6| = -(2x-6)$
$|x| = -x$
$|x+1| = x+1$

$-3(2x-6)-4x \geq x-(x+1)$

$-6x+18-4x \geq x-x-1$

$-10x \geq -19 \quad |:(-10)$

$x \leq \frac{19}{10}$ $\Rightarrow$ to splňují $\underline{\text{všechna } x \in \langle -1,0)}$

c) $\underline{|x \in \langle 0,3)|}$ $\Rightarrow$

$|2x-6| = -(2x-6)$
$|x| = x$
$|x+1| = x+1$

$-3(2x-6)+4x \geq x-(x+1)$

$-6x+18+4x \geq -1$

$-2x \geq -19$

$x \leq \frac{19}{2} = 9+\frac{1}{2}$ $\Rightarrow$ to splňují $\underline{\text{všechna } x \in \langle 0,3)}$

d) $\underline{|x \in \langle 3,\infty)|}$ $|2x-6| = 2x-6$; $|x| = x$; $|x+1| = x+1$

$3(2x-6)+4x \geq x-(x+1)$

$6x-18+4x \geq -1$

$10x \geq 17$

$x \geq \frac{17}{10} = 1{,}7$ $\Rightarrow$ to splňují $\underline{\text{všechna } x \in \langle 3,\infty)}$

$\Rightarrow \underline{\underline{x \in (-\infty,-1) \cup \langle -1,0) \cup \langle 0,3) \cup \langle 3,\infty) = \mathbb{R}}}$ (Nerovnost platí pro libovolné $x \in \mathbb{R}$)

4.) $|2x-6|-3 \geq -2|x-2|+1$

Nulové body:

$2x-6=0 \iff x=3$
$x-2=0 \iff x=2$

a) $\underline{x \in (-\infty,2)} \Rightarrow |2x-6| = -(2x-6)$
$|x-2| = -(x-2)$

$-(2x-6)-3 \geq 2(x-2)+1$
$-2x+6-3 \geq 2x-4+1$
$-2x+3 \geq 2x-3$
$-4x \geq -6$
$x \leq \frac{6}{4} = \frac{3}{2} = 1{,}5$ $\Rightarrow \underline{x \in (-\infty,\frac{3}{2}\rangle \cap (-\infty,2) = (-\infty,\frac{3}{2}\rangle}$

b) $\underline{x \in \langle 2,3)} \Rightarrow |2x-6| = -(2x-6)$
$|x-2| = x-2$

$-(2x-6)-3 \geq -2(x-2)+1$
$-2x+6-3 \geq -2x+4+1$
$3 \geq 5$ $\Rightarrow$ Nerovnici nesplňuje ani jedno x z intervalu $\langle 2,3)$

c) $\underline{x \in \langle 3,\infty)} \Rightarrow |2x-6| = 2x-6$
$|x-2| = x-2$

$2x-6-3 \geq -2(x-2)+1$
$2x-9 \geq -2x+5$
$4x-9 \geq 5$
$4x \geq 14$
$x \geq \frac{14}{4} = \frac{7}{2}$ $\Rightarrow \underline{x \in (\frac{7}{2},\infty) \cap \langle 3,\infty) = \langle \frac{7}{2},\infty)}$

$\Rightarrow$ Nerovnice je splněna pro libovolné $\underline{x \in (-\infty,\frac{3}{2}\rangle \cup \langle \frac{7}{2},\infty)}$

5) $\left|\frac{x-5}{x+1}\right| \leq 4$

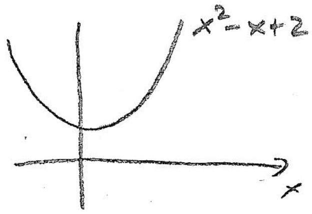
$x \neq -1$, n.b. $= \{5\}$

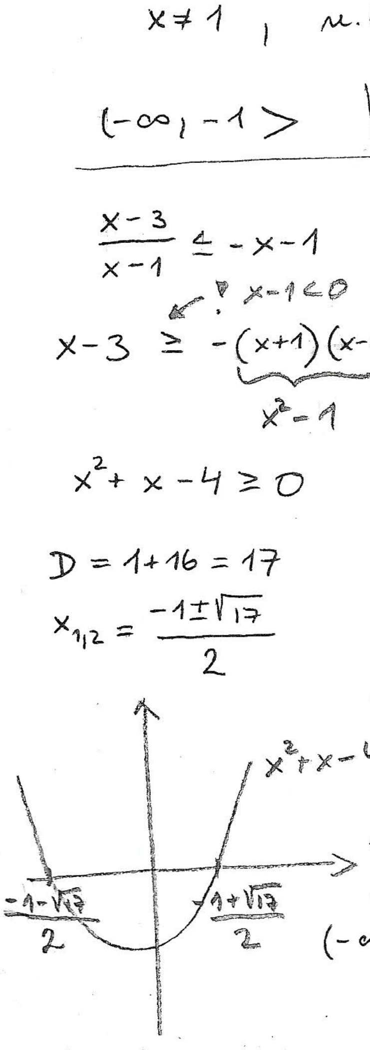
$(-\infty, -1)$ nebo	$(-1, 5\rangle$ nebo	$\langle 5, +\infty)$
$\frac{x-5}{x+1} \leq 4$! $x+1<0$ $x-5 \geq 4x+4$ $3x \leq -9$ $x \leq -3$ ↓ $(-\infty, -3\rangle$	$\frac{5-x}{x+1} \leq 4$! $x+1>0$ $5-x \leq 4x+4$ $5x \geq 1$ $x \geq \frac{1}{5}$ ↓ $\langle \frac{1}{5}, 5\rangle$	$\frac{x-5}{x+1} \leq 4$ ↓ analogicky jako 1. případ jen bez změny nerovn $x \geq -3$ ↓ $\langle 5, +\infty)$

Řešením je $(-\infty, -3\rangle \cup \langle \frac{1}{5}, +\infty)$.

6) $\frac{x-3}{x-1} \leq |x+1|$ (pracnější)

$x \neq 1$, n.b. $= \{-1\}$

$(-\infty, -1\rangle$	nebo $\langle -1, 1)$	nebo $(1, +\infty)$
$\frac{x-3}{x-1} \leq -x-1$! $x-1<0$ $x-3 \geq -\underbrace{(x+1)(x-1)}_{x^2-1}$ $x^2+x-4 \geq 0$ $D = 1+16 = 17$ $x_{1,2} = \frac{-1 \pm \sqrt{17}}{2}$	$\frac{x-3}{x-1} \leq x+1$ $x-3 \geq x^2-1$ $x^2-x+2 \leq 0$ $D = 1-8 = -7 < 0$  ↓ $\emptyset$	$\frac{x-3}{x-1} \leq x+1$ $x-3 \leq x^2-1$ $x^2-x+2 \geq 0$ ↓ $(1, +\infty)$



↓ $(-\infty, \frac{-1-\sqrt{17}}{2}\rangle$

$\Rightarrow$ Řešením je

$(-\infty, \frac{-1-\sqrt{17}}{2}\rangle \cup (1, +\infty)$.