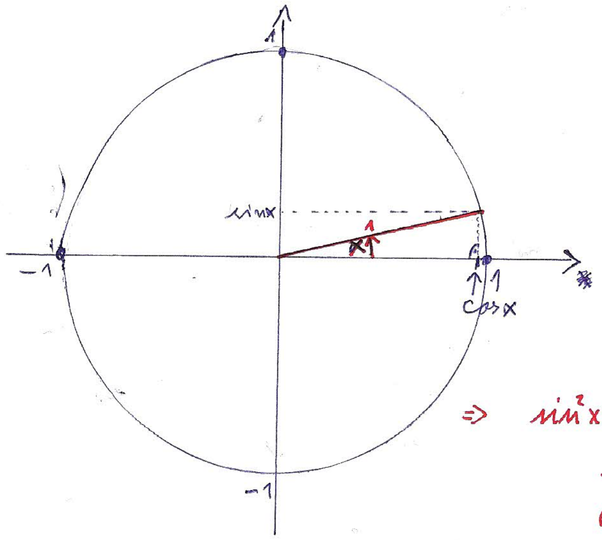
Goniometrické funkce

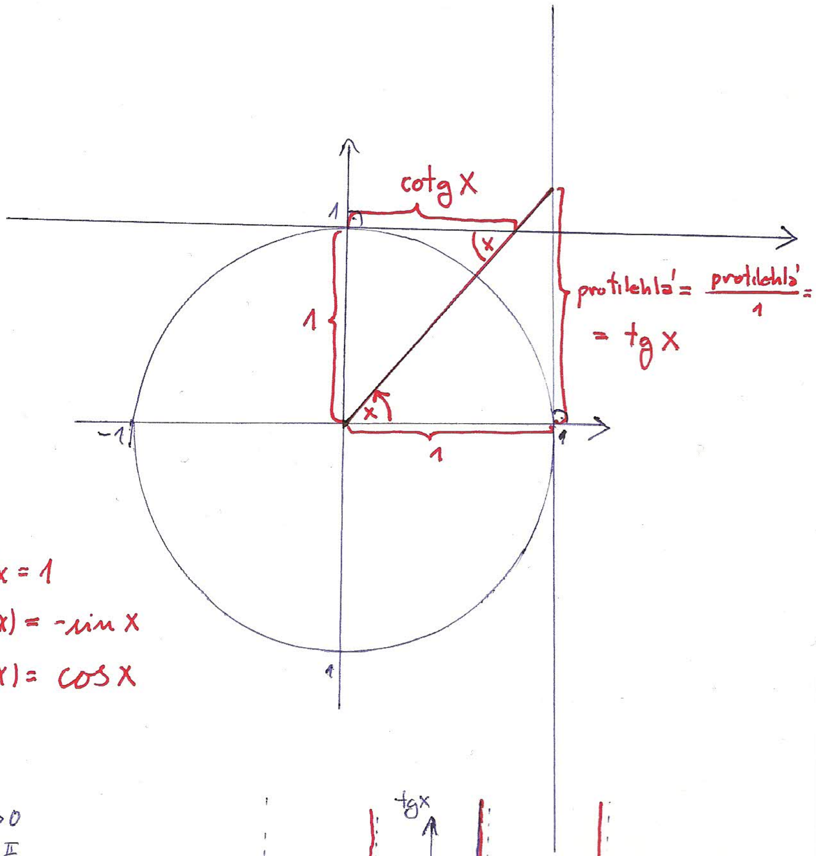
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\operatorname{cotg} x = \frac{\cos x}{\sin x}$$

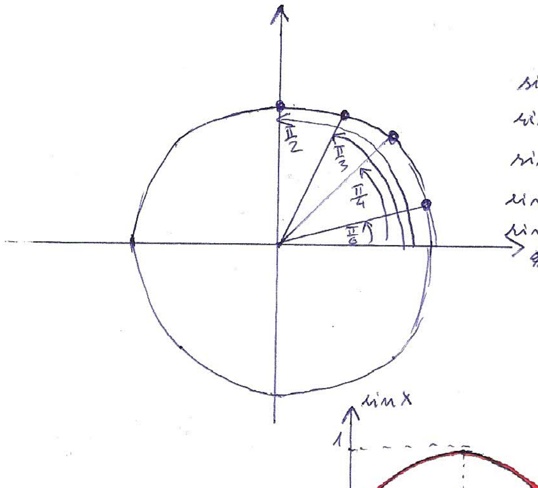




$\Rightarrow \quad \sin^2 x + \cos^2 x = 1$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$



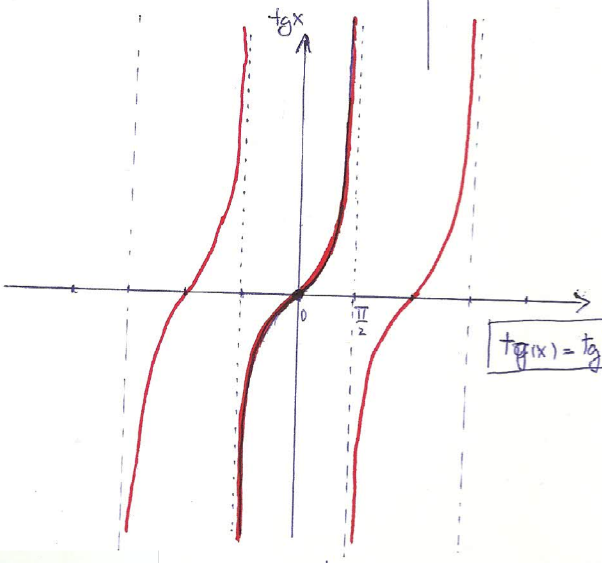
$\sin\frac{\pi}{2} = 1 = \frac{\sqrt{4}}{2} = \cos 0$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

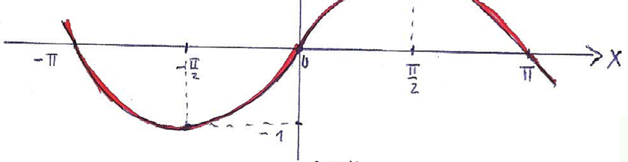
$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

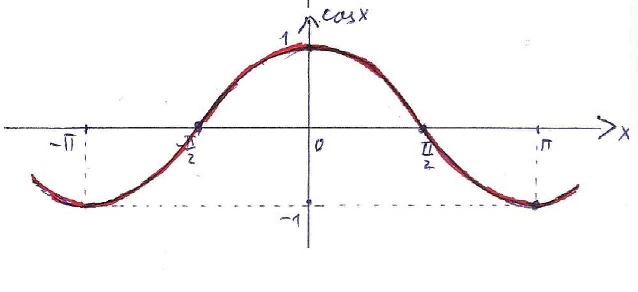
$\sin\frac{\pi}{6} = \frac{1}{2} = \cos\frac{\pi}{3}$

$\sin 0 = 0 = \cos\frac{\pi}{2}$



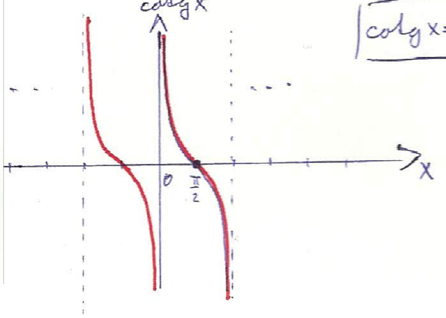
$\boxed{\operatorname{tg}(x) = \operatorname{tg}(x+\pi)}$





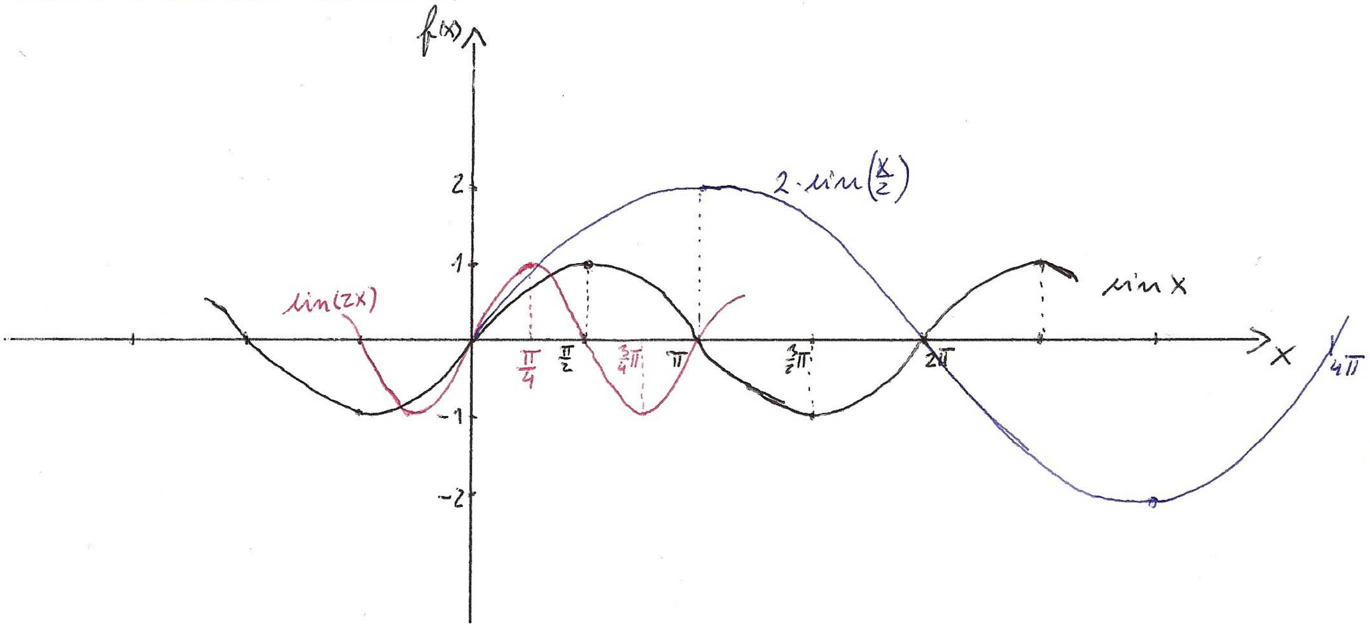
Definiční obory:

$f(x)$	D_f
$\sin x$	$\mathbb{R}$
$\cos x$	$\mathbb{R}$
$\operatorname{tg} x$	$\mathbb{R} - \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$
$\operatorname{cotg} x$	$\mathbb{R} - \{0 + k\pi \mid k \in \mathbb{Z}\}$

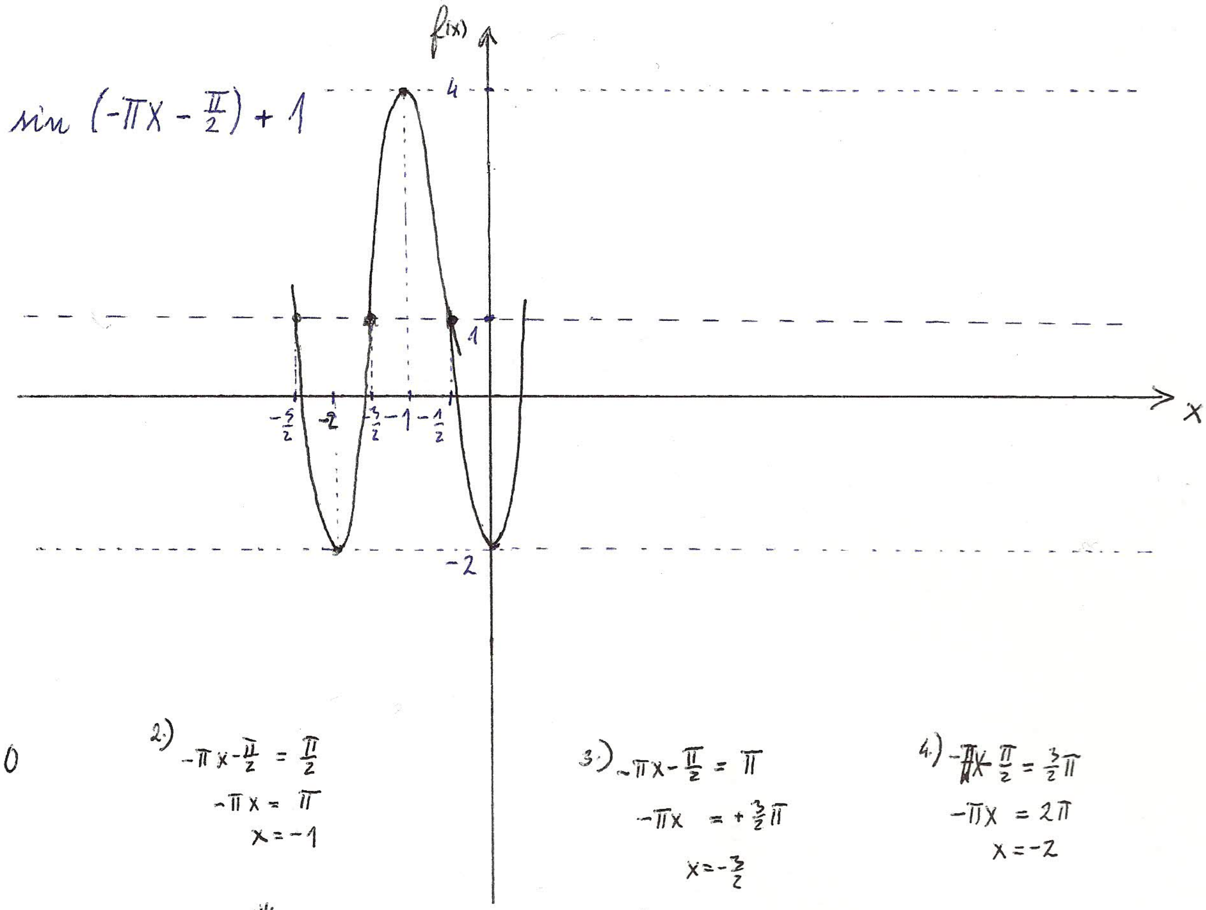


$\operatorname{cotg} x = \operatorname{cotg}(x+\pi)$

a) $f(x) = \sin x \ ; \ \sin(2x) \ ; 2\sin\left(\frac{x}{2}\right)$



b) $f(x) = 3 \cdot \sin\left(-\pi x - \frac{\pi}{2}\right) + 1$



1.) $-\pi x - \frac{\pi}{2} = 0$

$-\pi x = \frac{\pi}{2}$

$x = -\frac{1}{2}$

$\Downarrow$

$f(-\frac{1}{2}) = 3\sin 0 + 1 = 1$

2.) $-\pi x - \frac{\pi}{2} = \frac{\pi}{2}$

$-\pi x = \pi$

$x = -1$

$\Downarrow$

$f(-1) = 3\sin\frac{\pi}{2} + 1 = 4$

3.) $-\pi x - \frac{\pi}{2} = \pi$

$-\pi x = +\frac{3}{2}\pi$

$x = -\frac{3}{2}$

$\Downarrow$

$f(-\frac{3}{2}) = 3\sin\pi + 1 = 1$

4.) $-\pi x - \frac{\pi}{2} = \frac{3}{2}\pi$

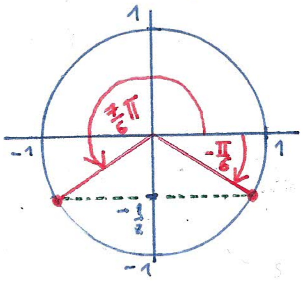
$-\pi x = 2\pi$

$x = -2$

$f(-2) = 3\sin\frac{3}{2}\pi + 1 = -3 + 1 = -2$

Př.: Nalezněte všechna $x \in \mathbb{R}$ splňující:

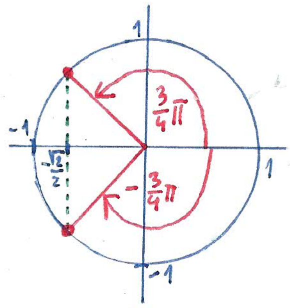
1.) $\sin 3X = -\frac{1}{2}$



$\Rightarrow$ $3X = -\frac{\pi}{6} + k2\pi \; /:3$, nebo $3X = \frac{7}{6}\pi + k\cdot 2\pi \; /:3$

$x = -\frac{\pi}{18} + k\frac{2\pi}{3}$; $k\in\mathbb{Z}$, nebo $x = \frac{7}{18}\pi + k\frac{2\pi}{3}$; $k\in\mathbb{Z}$

2.) $\cos(5X+1) = -\frac{\sqrt{2}}{2}$



$\Rightarrow$

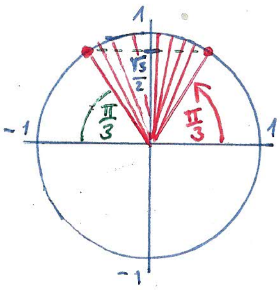
$5X+1 = \frac{3}{4}\pi + k2\pi$ nebo $5X+1 = -\frac{3}{4}\pi + k2\pi$

$5X = \frac{3}{4}\pi - 1 + k2\pi$ nebo $5X = -\frac{3}{4}\pi - 1 + k2\pi$

$X = \frac{3}{20}\pi - \frac{1}{5} + k\frac{2\pi}{5}, k\in\mathbb{Z}$ nebo $X = -\frac{3}{20}\pi - \frac{1}{5} + k\frac{2\pi}{5}, k\in\mathbb{Z}$

3.) $\sin(2X) \geq \frac{\sqrt{3}}{2}$ $\Rightarrow$ $2X \in \left\langle \frac{\pi}{3} + k2\pi, \frac{2\pi}{3} + k2\pi \right\rangle$ $/:2$

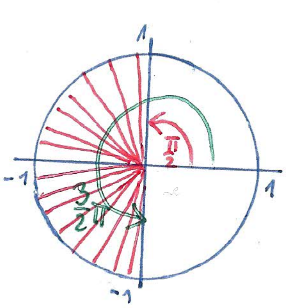
$X \in \left\langle \frac{\pi}{6} + k\cdot\pi, \frac{2\pi}{6} + k\pi \right\rangle, k\in\mathbb{Z}$



4.) $\cos(1-x) < 0$ $\Rightarrow$ $\frac{\pi}{2} + k2\pi < 1-x < \frac{3}{2}\pi + k2\pi$

$\frac{\pi}{2} - 1 + k2\pi < -x < \frac{3}{2}\pi - 1 + k2\pi$ $/\cdot(-1)$

$1 - \frac{\pi}{2} - k2\pi > x > 1 - \frac{3}{2}\pi - k2\pi$



$x \in \left(1 - \frac{3}{2}\pi - k2\pi; 1 - \frac{\pi}{2} - k2\pi\right); k\in\mathbb{Z}$

Goniometrické rovnice

Př. Vyřešte rovnice

1) $\boxed{2\,\mathrm{tg}\,x\,\sin 3x = \sqrt{3}\,\mathrm{tg}\,x}$ $\Rightarrow$ $x \neq \frac{\pi}{2} + k\pi$ (tam není $\mathrm{tg}\,x$ definován)

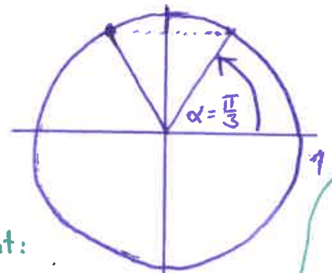
I.) $\mathrm{tg}\,x \neq 0 \Rightarrow$ můžeme rovnici podělit:

$$2\,\mathrm{tg}\,x \sin 3x = \sqrt{3}\,\mathrm{tg}\,x \quad /:\mathrm{tg}\,x$$

$$2\sin 3x = \sqrt{3}$$

$$\sin 3x = \frac{\sqrt{3}}{2} \qquad /\alpha = 3x$$

$$\sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow$$



$$\alpha = \begin{cases} \frac{\pi}{3} + k_1 2\pi \\ \frac{2\pi}{3} + k_2 2\pi \end{cases}$$

$$\Downarrow$$

$$x = \begin{cases} \frac{\pi}{9} + k_1\frac{2}{3}\pi \neq \frac{\pi}{2} + k\pi \\ \frac{2\pi}{9} + k_2\frac{2}{3}\pi \neq \frac{\pi}{2} + k\pi \end{cases}$$

Musíme ověřit:

$\frac{\pi}{9} + k_1\frac{2}{3}\pi \overset{?}{=} \frac{\pi}{2} + k\pi \;/\cdot 18$

$2\pi + k_1 12\pi = 9\pi + k\cdot 18\pi$

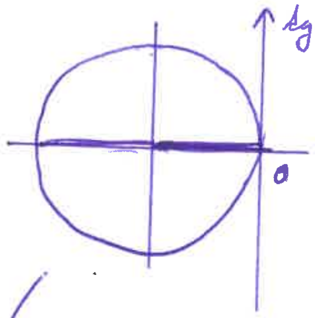
$\underbrace{(2k_1 - 3k)}_{\in\mathbb{Z}} 6\pi = 7\pi \Rightarrow$ spor!

analogicky

II.) $\mathrm{tg}\,x = 0 \Rightarrow$ rovnice je splněna

$\mathrm{tg}\,x = 0 \Leftrightarrow \underline{x = k\cdot\pi}$

$\frac{\pi}{2} + k\cdot\pi$



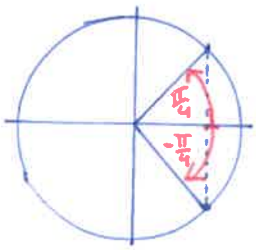
$$x = \begin{cases} k\cdot\pi, & k\in\mathbb{Z} \\ \frac{\pi}{9} + k_1\frac{2}{3}\pi, & k_1\in\mathbb{Z} \\ \frac{2\pi}{9} + k_2\frac{2}{3}\pi, & k_2\in\mathbb{Z} \end{cases}$$

2.) $\boxed{2\sin x\,\cos(5x) = \sqrt{2}\,\sin x}$

I.) $\sin x \neq 0 \Rightarrow$ podělíme $\Rightarrow$

$$2\cos(5x) = \sqrt{2}$$

$$\cos(5x) = \frac{\sqrt{2}}{2}$$



$$\Rightarrow 5x = \begin{cases} \frac{\pi}{4} + k2\pi \\ -\frac{\pi}{4} + k2\pi \end{cases}$$

II.) $\sin x = 0 \Rightarrow$ rovnice je splněna

$\sin x = 0 \Leftrightarrow \underline{x = k\cdot\pi}$

$$\Rightarrow x = \begin{cases} \frac{\pi}{20} + k\frac{2\pi}{5}, & k\in\mathbb{Z} \\ -\frac{\pi}{20} + k\frac{2\pi}{5}, & k\in\mathbb{Z} \\ k\cdot\pi, & k\in\mathbb{Z} \end{cases}$$

3.) $\boxed{\sin^2(3x) - \sin(3x) = 0}$ $\quad / \ t = \sin(3x)$

$$t^2 - t = 0$$

$$t\,(t-1) = 0 \quad \Rightarrow \quad t = \begin{cases} 0 = \sin(3x) \\ \text{nebo} \\ 1 = \sin(3x) \end{cases}$$

$\Rightarrow$ I.) $\sin(3x) = 0$ nebo II.) $\sin(3x) = 1$

$3x = k\pi, \ k \in \mathbb{Z}$ $\qquad$ $3x = \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$

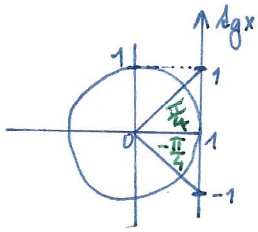
$\underline{\underline{x = k\frac{\pi}{3}; \ k \in \mathbb{Z}, \ \text{nebo} \quad x = \frac{\pi}{6} + k\frac{\pi}{3}; \ k \in \mathbb{Z}}}$

4.) $\boxed{tg^2 x + (1+\sqrt{3})\, tg\, x + \sqrt{3} = 0}$

Má smysl řešit pouze pro $x \in D(tg) \Rightarrow x \neq \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$

$(tg\, x + 1)(tg\, x + \sqrt{3}) = 0$

$$tg\, x = \begin{cases} -1 \\ -\sqrt{3} \end{cases}$$



; $\quad tg\left(-\frac{\pi}{3}\right) = \frac{\sin\left(-\frac{\pi}{3}\right)}{\cos\left(-\frac{\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\Rightarrow \quad x = -\frac{\pi}{4} + k_1\pi, \ k_1 \in \mathbb{Z}$ nebo $x = -\frac{\pi}{3} + k_2\pi, \ k_2 \in \mathbb{Z}$

Musíme ale zkontrolovat, zda taková x nálezí do $D(tg)$. Tzn. nestane se náhodou, že:

$x = -\frac{\pi}{4} + k_1\pi = \frac{\pi}{2} + k\pi \ \big| \cdot \frac{4}{\pi}$, nebo $\qquad x = -\frac{\pi}{3} + k_2\pi = \frac{\pi}{2} + k\pi \ \big| \cdot \frac{6}{\pi}$?

$\Downarrow \qquad\qquad\qquad \Downarrow$

$-1 + 4k_1 = 2 + 4k \qquad\qquad -2 + 6k_2 = 3 + 6k$

$4k_1 - 4k = 3 \qquad\qquad 6k_2 - 6k = 5$

násobek 4 $\Rightarrow$ spor! $\qquad$ násobek 6 $\Rightarrow$ spor!

$\Rightarrow$ To se nikdy nestane $\Rightarrow$

$\underline{\underline{x = -\frac{\pi}{4} + k_1\pi; \ k_1 \in \mathbb{Z}, \ \text{nebo} \quad x = -\frac{\pi}{3} + k_2\pi; \ k_2 \in \mathbb{Z}}}$

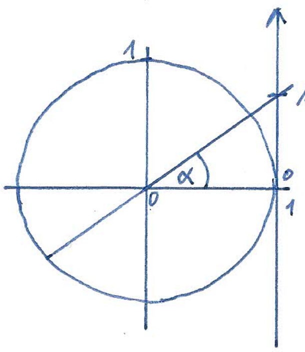
5.) $\boxed{\operatorname{tg}(7x) = \operatorname{tg}(5x)}$

Má smysl řešit pouze pro x taková, že $7x$ i $5x \in D(\operatorname{tg})$,

tzn. musí být splněno:

$7x \neq \frac{\pi}{2} + k_1\pi$ a $5x \neq \frac{\pi}{2} + k_2\pi$, kde $k_1, k_2 \in \mathbb{Z}$ $\Rightarrow$

$x \neq \frac{\pi}{14} + k_1\frac{\pi}{7}$ a $x \neq \frac{\pi}{10} + k_2\frac{\pi}{5}$



$\operatorname{tg}\alpha_1 = \operatorname{tg}\alpha_2 \Leftrightarrow (\alpha_1 = \alpha_2 + k\pi, k \in \mathbb{Z} \wedge \alpha_1, \alpha_2 \in D(\operatorname{tg}))$

$\Rightarrow (7x) = (5x) + k\pi$

$2x = k\pi$

$\underline{x = k\frac{\pi}{2}}$ je řešením?

Zjišťujeme, zda pro nějaké k najdeme $k_1, k_2 \in \mathbb{Z}$ takové, že:

$k\frac{\pi}{2} = \frac{\pi}{14} + k_1\frac{\pi}{7}$ $/ \cdot \frac{14}{\pi}$ nebo $k\frac{\pi}{2} = \frac{\pi}{10} + k_2\frac{\pi}{5}$ $/ \cdot \frac{10}{\pi}$

$7k = 1 + 2k_1$ $\qquad$ $5k = 1 + 2k_2$

$2k_1 = 7k - 1$ $\qquad$ $2k_2 = 5k - 1$

$k_1 = \frac{7k-1}{2}$ $\in \mathbb{Z}$ pouze pro k-liché $\qquad$ $k_2 = \frac{5k-1}{2}$ $\in \mathbb{Z}$ pouze pro k-liché

$\Rightarrow$ Pro k-liché platí, že $k\frac{\pi}{2}$ není řešením a pro $k = 2r$, kde $r \in \mathbb{Z}$

je $k\frac{\pi}{2} = \underbrace{2r\frac{\pi}{2}}_{r \cdot \pi}$ řešením. $\Rightarrow$

$$\underline{\underline{x = r \cdot \pi, \text{ kde } r \in \mathbb{Z}}}$$

Př. Vyřešte

1.) $-2\cos^2 x - 3\sin x + 3 = 0 \qquad / \quad \cos^2 x = 1 - \sin^2 x$

$-2(1-\sin^2 x) - 3\sin x + 3 = 0$

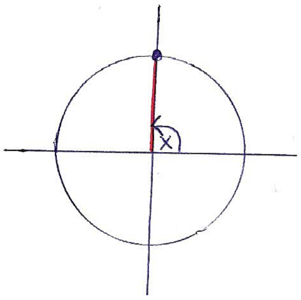
$2\sin^2 x - 3\sin x + 1 = 0 \qquad / \quad s = \sin x$

$2s^2 - 3s + 1 = 0$

$$s_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

a) $\boxed{s = 1}$

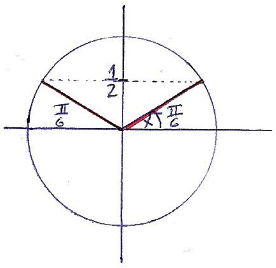
$\sin x = 1$



$\Rightarrow \underline{x = \frac{\pi}{2} + k \cdot 2\pi, \; k \in \mathbb{Z}}$

b) $\boxed{s = \frac{1}{2}}$

$\sin x = \frac{1}{2}$



$\Rightarrow$

$\underline{x = \frac{\pi}{6} + k \cdot 2\pi, \; k \in \mathbb{Z}}$

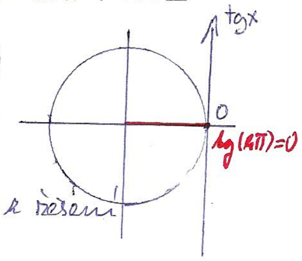
$\underline{x = \frac{5}{6}\pi + k \cdot 2\pi, \; k \in \mathbb{Z}}$

$\Leftrightarrow \underline{\underline{x \in \left\{ \frac{\pi}{2} + k \cdot 2\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{6} + k \cdot 2\pi \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{5}{6}\pi + k \cdot 2\pi \mid k \in \mathbb{Z} \right\}}}$

2.) $2\sin x = \sqrt{3}\,\mathrm{tg}\,x$

$2\sin x = \sqrt{3}\,\dfrac{\sin x}{\cos x} \quad /: \sin x \neq 0$, ale $\sin x = 0 \iff x = k\pi,\ k\in\mathbb{Z}$

$\underbrace{2\cdot\sin(k\pi)}_{0} = \underbrace{\sqrt{3}\,\mathrm{tg}(k\pi)}_{0}$



$2 = \sqrt{3}\,\dfrac{1}{\cos x} \quad /\cdot \cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi = \left\langle \begin{matrix} \frac{\pi}{2} + k^{*}2\pi \\ \frac{3}{2}\pi + k^{*}2\pi \end{matrix} \right.$

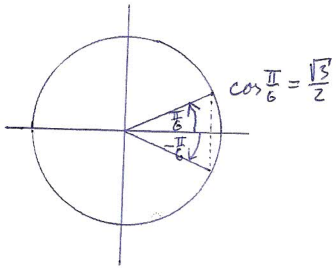
$(x = \frac{\pi}{2} + k\pi$ není řešením$)$

$\Rightarrow \underline{x = k\cdot\pi,\ k\in\mathbb{Z}}$ je jedním z řešení

$\neq \frac{\pi}{2} + k\pi$

$2\cos x = \sqrt{3}$

$\cos x = \dfrac{\sqrt{3}}{2}$



$\Rightarrow \underline{x = \dfrac{\pi}{6} + k2\pi,\ k\in\mathbb{Z}}$

$\underline{x = -\dfrac{\pi}{6} + k2\pi,\ k\in\mathbb{Z}}$

$\neq \frac{\pi}{2} + k\pi$

$\Leftrightarrow \underline{\underline{x \in \{k\pi \mid k\in\mathbb{Z}\} \cup \{\frac{\pi}{6} + k2\pi \mid k\in\mathbb{Z}\} \cup \{-\frac{\pi}{6} + k2\pi \mid k\in\mathbb{Z}\}}}$

3.) $\cos^2 x - 5\sin x + 5 = 0$

$1 - \sin^2 x - 5\sin x + 5 = 0$

$-\sin^2 x - 5\sin x + 6 = 0$

$\sin^2 x + 5\sin x - 6 = 0 \quad /\ s = \sin x$

~~$(\sin x + 2)(\sin x$~~

$s^2 + 5s - 6 = 0 \quad \Rightarrow \quad s_{1,2} = \dfrac{-5 \pm \sqrt{25+24}}{2} = \dfrac{-5 \pm 7}{2} = \left\langle \begin{matrix} -\frac{12}{2} = -6 \Rightarrow \text{žádné řešení! } \sin x \neq -6 \\ \frac{2}{2} = 1 = \sin x \end{matrix} \right.$

$\Rightarrow \underline{\underline{x = \frac{\pi}{2} + k2\pi,\ k\in\mathbb{Z}}}$

Př 2.: $\cos^2 x - 5\sin x + 5 = 0$

$1 - \sin^2 x - 5\sin x + 5 = 0$

$-\sin^2 x - 5\sin x + 6 = 0 \quad |\,\text{subst: } s = \sin x$

$-s^2 - 5s + 6 = 0$

$$s_{1,2} = \frac{5 \pm \sqrt{25+24}}{-2} = \frac{5\pm 7}{-2} = \begin{cases} \frac{12}{-2} = -6 = \sin x \Rightarrow \text{žádné řešení} \\ \frac{-2}{-2} = 1 = \sin x \Rightarrow \underline{\underline{x = \frac{\pi}{2} + 2k\pi,\ k\in\mathbb{Z}}} \end{cases}$$

Př 3.: $\cos(2x) + \sin x = 1$

$\left((\cos x + i\sin x)^2 = \cos 2x + i\sin 2x = \cos^2 x + i2\cos x\sin x - \sin^2 x \right)$

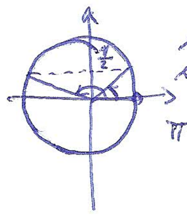
$\cos^2 x - \sin^2 x + \sin x = 1$

$1 - 2\sin^2 x + \sin x = 1$

$-2\sin^2 x + \sin x = 0$

$\sin x(-2\sin x + 1) = 0$

$$\Rightarrow \sin x = \begin{cases} 0 \Rightarrow x = \begin{cases} \underline{\underline{0 + 2k\pi,\ k\in\mathbb{Z}}} \\ \underline{\underline{\pi + 2k\pi,\ k\in\mathbb{Z}}} \end{cases} \\ \frac{1}{2} \Rightarrow x = \begin{cases} \underline{\underline{\frac{\pi}{6} + 2k\pi,\ k\in\mathbb{Z}}} \\ \underline{\underline{\frac{5}{6}\pi + 2k\pi,\ k\in\mathbb{Z}}} \end{cases} \end{cases}$$



$\sin 0 = 0$

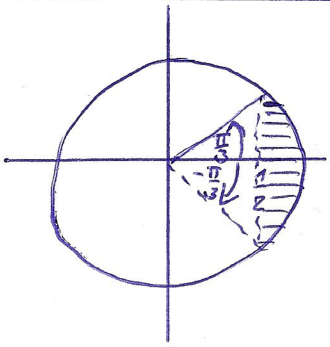
$\sin\frac{\pi}{6} = \frac{1}{2}$

$\pi - \frac{\pi}{6} = \frac{5}{6}\pi$

Goniometrické nerovnice

Př.: Vyřešte nerovnice

1) $\cos 3x > \frac{1}{2}$



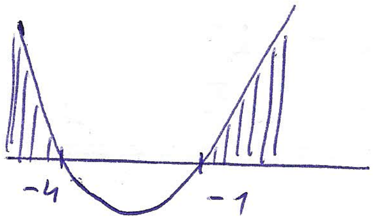
$\Rightarrow \quad 3x \in \left(-\frac{\pi}{3}+k2\pi,\ \frac{\pi}{3}+k2\pi\right)$

$\Rightarrow \quad \underline{\underline{x \in \left(-\frac{\pi}{9}+k\frac{2}{3}\pi\ ;\ \frac{\pi}{9}+k\frac{2}{3}\pi\right),\ k\in\mathbb{Z}}}$

2.) $\sin^2 x + 5\sin x + 4 \geq 0$ subst.: $s = \sin x$

$s^2 + 5s + 4 \geq 0$

$s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = \frac{-5\pm\sqrt{9}}{2} = \begin{cases} -4 \\ -1 \end{cases}$



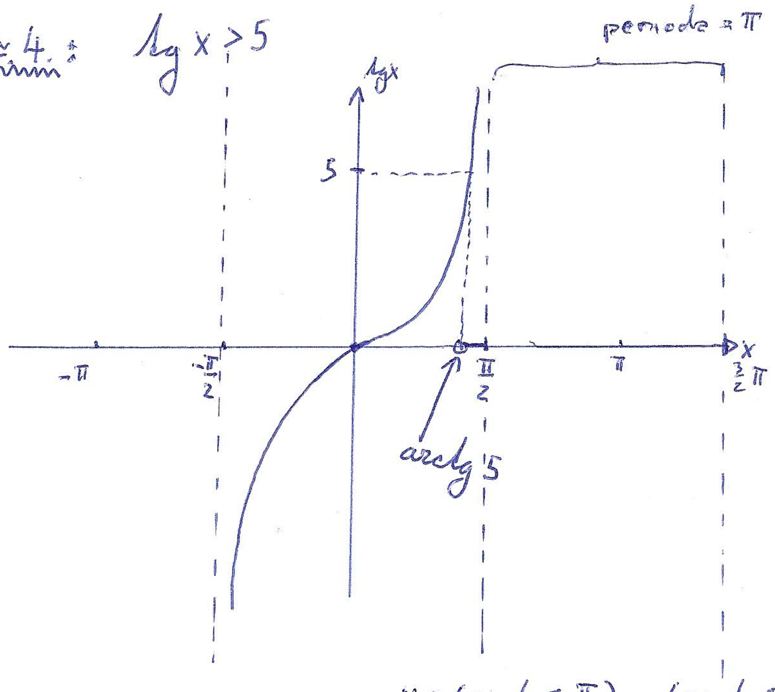
$\Leftrightarrow \quad s \in (-\infty, -4\rangle \cup \langle -1, \infty) \quad \Leftrightarrow$

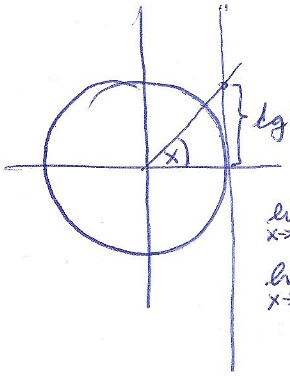
$\Leftrightarrow \quad \sin x \in (-\infty, -4\rangle \cup \langle -1, \infty) \quad \Rightarrow$

$\Rightarrow$ Vzhledem k tomu, že $\sin x \in \langle -1, 1\rangle$, je nerovnost splněna pro libovolné $x \in \mathbb{R}$.

$\sin^2 x + 2{,}5\sin x + 1 = 0$

Př 4.: $\operatorname{tg} x > 5$





$\Rightarrow \operatorname{tg} 0 = 0$

$\lim\limits_{x \to \frac{\pi}{2}^-} \operatorname{tg} x = \infty$

$\lim\limits_{x \to -\frac{\pi}{2}^+} \operatorname{tg} x = -\infty$

$x \in (\operatorname{arctg} 5, \frac{\pi}{2}) \cup (\operatorname{arctg} 5 + \pi, \frac{\pi}{2} + \pi) \cup (\operatorname{arctg} 5 + 2\pi, \frac{\pi}{2} + 2\pi) \cup \ldots$

$$\Rightarrow \quad \underline{\underline{x \in \bigcup_{k \in \mathbb{Z}} (\operatorname{arctg} 5 + k\pi \; ; \; \frac{\pi}{2} + k \cdot \pi)}}$$

Př 5:

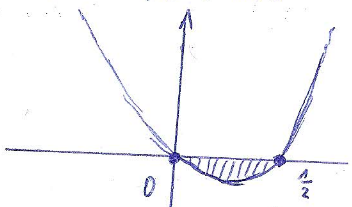
$\sin^2 x - \cos^2 x - \sin x + 1 < 0$

$\sin^2 x - (1 - \sin^2 x) - \sin x + 1 < 0$

$2 \sin^2 x - \sin x < 0 \qquad / t = \sin x$

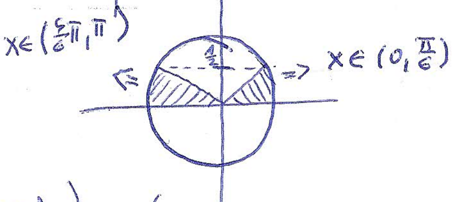
$2t^2 - t < 0$

$t_{1,2} = \left\langle \begin{array}{l} 0 \\ \frac{1}{2} \end{array} \right.$



$\Rightarrow \quad t \in (0, \frac{1}{2})$

$\sin x \in (0, \frac{1}{2})$



$$\Rightarrow \underline{\underline{x \in \left(\bigcup_{k \in \mathbb{Z}} (0 + 2k\pi \; ; \; \frac{\pi}{6} + 2k\pi) \right) \cup \left(\bigcup_{k \in \mathbb{Z}} (\frac{5}{6}\pi + 2k\pi \; ; \; \pi + 2k\pi) \right)}}$$

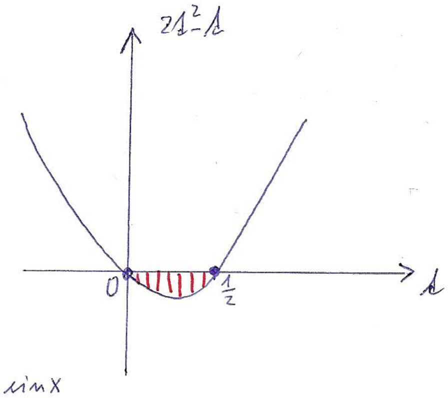
4.) $\sin^2 x - \cos^2 x - \sin x + 1 < 0$

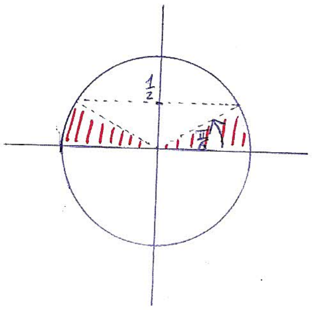
$\sin^2 x - (1 - \sin^2 x) - \sin x + 1 < 0$

$2\sin^2 x - \sin x < 0 \qquad | \; s = \sin x$

$2s^2 - s < 0$

$s(2s - 1) < 0 \iff s \in (0, \frac{1}{2}) \ni \sin x$





$\Rightarrow \; x \in (0, \frac{\pi}{6}) \cup (0 + 2\pi, \frac{\pi}{6} + 2\pi) \cup (0 + 2\cdot 2\pi, \frac{\pi}{6} + 2\cdot 2\pi) \cup \ldots = \bigcup_{k\in\mathbb{Z}} (0 + k2\pi, \frac{\pi}{6} + k2\pi)$

nebo

$x \in (\frac{5}{6}\pi, \pi) \cup (\frac{5}{6}\pi + 2\pi, \pi + 2\pi) \cup (\frac{5}{6}\pi + 2\cdot 2\pi, \pi + 2\cdot 2\pi) \cup \ldots = \bigcup_{k\in\mathbb{Z}} (\frac{5}{6}\pi + k2\pi, \pi + k2\pi)$

$\iff \; x \in \left(\bigcup_{k\in\mathbb{Z}} (0 + k2\pi, \frac{\pi}{6} + k2\pi) \right) \cup \left(\bigcup_{k\in\mathbb{Z}} (\frac{5}{6}\pi + k2\pi, \pi + k2\pi) \right)$

5.) $\sin x + \cos(2x) \geq 0$ $\qquad$ | Určíme $\cos(2x)$: $\quad (\cos x + i \sin x)^2 = \cos(2x) + i \sin(2x) =$

$= \cos^2 x + 2i \cos x \sin x - \sin^2 x$

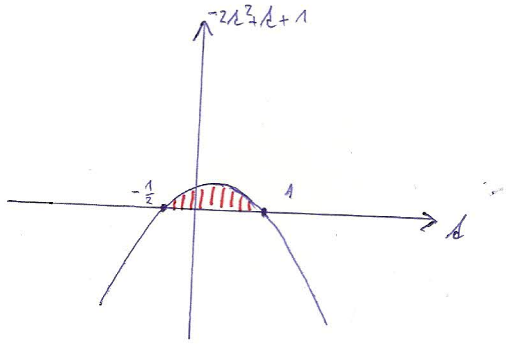
$\sin x + \cos^2 x - \sin^2 x \geq 0$

$\sin x + 1 - \sin^2 x - \sin^2 x \geq 0$

$-2\sin^2 x + \sin x + 1 \geq 0 \qquad | \; s = \sin x$

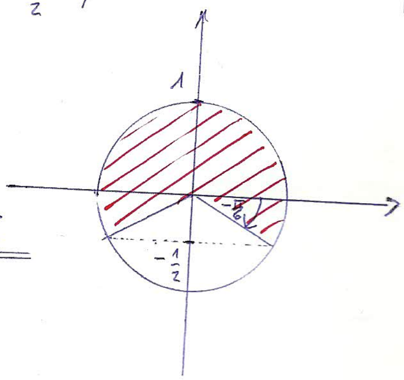
$-2s^2 + s + 1 \geq 0$

$\left(s_{1,2} = \frac{-1 \pm \sqrt{1+8}}{-4} = \frac{-1 \pm 3}{-4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \right)$



$\iff s = \sin x \in \langle -\frac{1}{2}, 1 \rangle$

$\iff x \in \langle -\frac{\pi}{6} + k2\pi, \frac{7}{6}\pi + k2\pi \rangle, \; k \in \mathbb{Z}$



Jak si (ne)pamatovat součtové vzorce

(P)

1.) $\sin(3+4) \neq \sin 3 + \sin 4$

2.) $\sin(3\cdot 4) \neq \sin 3 \cdot \sin 4$

Součtové vzorce:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

Využijeme vlastnosti násobení komplexních čísel v goniometrickém tvaru:

$$(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$$

ale také: $(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta + i(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

Jde o totéž komplexní číslo => musí se rovnat jeho reálné části i jeho imag. části.

Př: Vyřešte rovnici $\cos(x-\frac{\pi}{2}) + \sin x = 1$

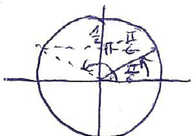
$$\sin x + \sin x = 1$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$

$/ \cos(x-\frac{\pi}{2}) = \cos x\cos(-\frac{\pi}{2}) - \sin x \sin(-\frac{\pi}{2}) =$

$\alpha = x$, $\beta = -\frac{\pi}{2}$

$= \cos x \underbrace{\cos\frac{\pi}{2}}_{=0} - \sin x\,\underbrace{(-\sin\frac{\pi}{2})}_{=-1} =$

$= \sin x$



$$\Rightarrow x = \begin{cases} \frac{\pi}{6} + k\cdot 2\pi, & k\in\mathbb{Z} \\ \pi - \frac{\pi}{6} + k2\pi = \frac{5}{6}\pi + k2\pi, & k\in\mathbb{Z} \end{cases}$$

Vzorce pro součet goniometrických funkcí

(!)

$$\sin x + \sin y \neq \sin(x+y)$$

$$\cos 3 + \cos 2 \neq \cos 5$$

Vzorce pro součet sin-ů a cos-inů:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2}$$

Př: Vyřešte rovnici:

$$\sin(5x) = \sin\left(3x - \frac{\pi}{2}\right)$$

$$\sin(5x) - \sin\left(3x - \frac{\pi}{2}\right) = 0$$

$$2 \sin \frac{5x - (3x - \frac{\pi}{2})}{2} \cos \frac{5x + (3x - \frac{\pi}{2})}{2} = 0 \quad /:2$$

$$\sin \frac{2x + \frac{\pi}{2}}{2} \cos \frac{8x - \frac{\pi}{2}}{2} = 0$$

$$\sin\left(x + \frac{\pi}{4}\right) \cos\left(4x - \frac{\pi}{4}\right) = 0 \quad \Rightarrow$$

buď $\boxed{\sin\left(x + \frac{\pi}{4}\right) = 0}$ a nebo $\boxed{\cos\left(4x - \frac{\pi}{4}\right) = 0}$

$\Updownarrow$

$x + \frac{\pi}{4} = k \cdot \pi$, kde $k \in \mathbb{Z}$

$\underline{x = -\frac{\pi}{4} + k\pi}$, $k \in \mathbb{Z}$

$4x - \frac{\pi}{4} = \frac{\pi}{2} + k \cdot \pi$, $k \in \mathbb{Z}$

~~$x = \frac{\pi}{16} + k\frac{\pi}{4}$, $k \in \mathbb{Z}$~~

$x - \frac{\pi}{16} = \frac{\pi}{8} + k\frac{\pi}{4}$

$x = \frac{\pi}{16} + \frac{2\pi}{16} + k\frac{\pi}{4}$

$\underline{x = \frac{3\pi}{16} + k\frac{\pi}{4}}$, $k \in \mathbb{Z}$

$$\Rightarrow \quad x \in \bigcup_{k \in \mathbb{Z}} \left\{ -\frac{\pi}{4} + k\pi ; \ \frac{3\pi}{16} + k\frac{\pi}{4} \right\}$$