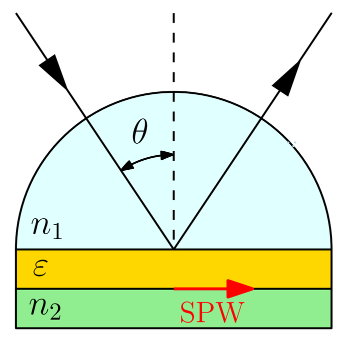
# Measurement of the dispersion of a liquid analyte using surface plasmon resonance: a theoretical approach

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## Abstract

A theoretical study of a spectral method based on surface plasmon resonance (SPR) to measure the dispersion of a liquid analyte is presented. Using the material dispersion of the SPR structure and the analyte, the resonance wavelength for the ratio of the reflectances for $p$- and $s$-polarized waves is determined for different angles of incidence. These theoretical values are utilized in obtaining the refractive index of the analyte as a function of the resonance wavelength using two procedures. The dispersion of the analyte thus retrieved is compared with the one used in the model and it is concluded that one of the procedures is more accurate than the other. The applicability of the new method is demonstrated for two different analytes, water and ethanol, and the measurement range is specified.

## 1. Theoretical model

An ideal experiment setup of the Kretschmann configuration [1, 2] with an optical prism and a four-layer system as an SPR structure is considered, where the surface plasmons are excited in the metallic film by an attenuated total reflection. The SPR structure used in the theoretical model consists of four layers: an SF10 glass plate, an adhesion chromium layer of 2 nm thickness, a gold layer of 44.8 nm thickness, and an analyte (water or ethanol).

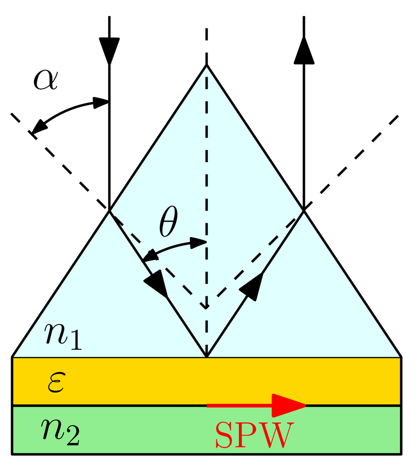


**Figure 1:** *The schematic drawing of the Kretschmann configuration with an equilateral coupling prism.*

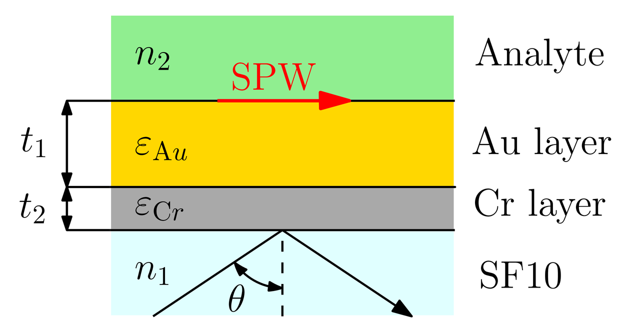


**Figure 2:** *The schematic drawing of four-layer SPR structure used in theoretical model.*

## 2. Theoretical results and discussion

When the parallel component of the propagation constant of the incident light wave is the same as the real part of the surface plasmon wave propagation constant, the incident light wave resonantly excites the SPW. The power carried by reflected wave drops down, which will result in a drop of spectral reflectance power (see Figs. 3 and 4), and a minimum (dip) can be observed at a specific resonance wavelength (see Figs. 5 and 6) corresponding to a specific angle of incidence on the dielectric-metal boundary. Using two procedures, theoretical values of the ratio of reflectances for $p$- and $s$-polarized waves at different angles of incidence are utilized in obtaining the refractive index of the analyte as a function of the resonance wavelength. The dispersion of the analyte (water and ethanol) thus obtained in a range of 400-900 nm is compared with the dependence used in the model.

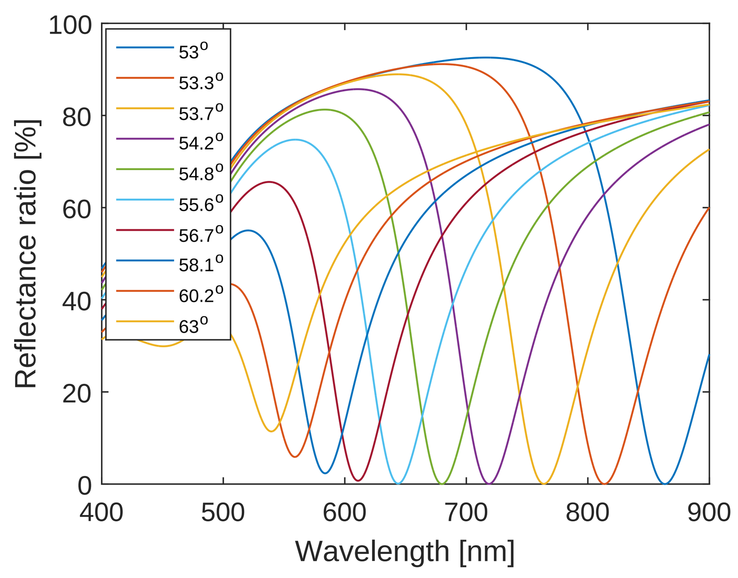


**Figure 3:** *Theoretical spectral dependencies of the reflectance ratio $R_p(\lambda)/R_s(\lambda)$ for different angles of incidence (water).*

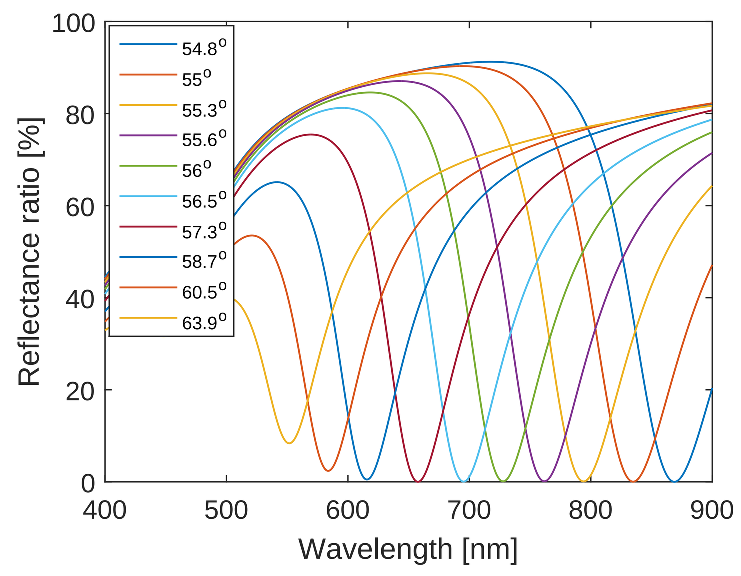


**Figure 4:** *Theoretical spectral dependencies of the reflectance ratio $R_p(\lambda)/R_s(\lambda)$ for different angles of incidence (ethanol).*

Increasing the angle of incidence results in a slower change of the resonance wavelength and the precision of the extraction of refractive index values is not constant in whole spectral range and changes accordingly to this dependence.

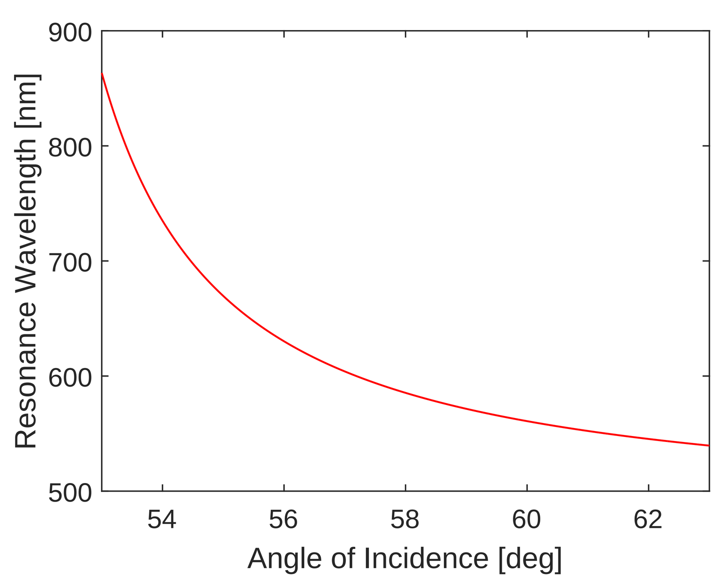


**Figure 5:** *Resonance wavelength as a function of the angle of incidence on the dielectric-metal boundary (water).*

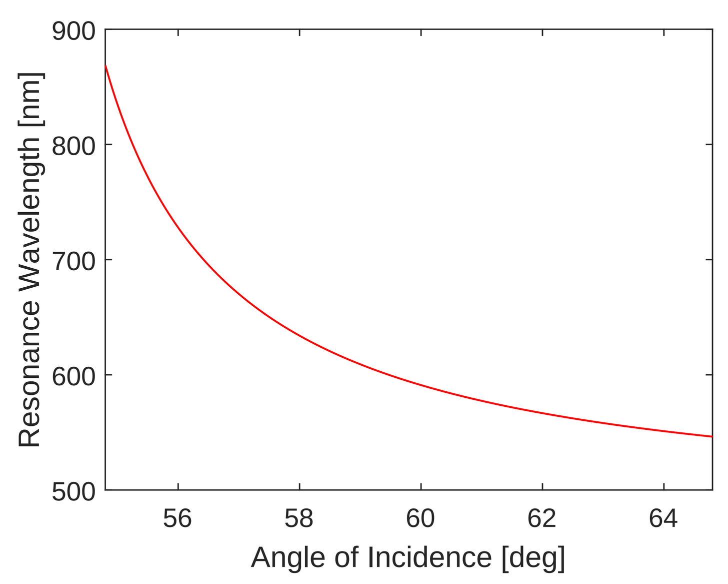


**Figure 6:** *Resonance wavelength as a function of the angle of incidence on the dielectric-metal boundary (ethanol).*

The first procedure utilizes the dependence of the reflectance ratio on the refractive index of the analyte at different resonance wavelengths. This dependence is similar to the spectral dependence since a minimum (dip) can also be observed from which the refractive index is extracted. The procedure provides results immediately. However, for resonance wavelengths that change with increasing angle of incidence very slowly, the accuracy significantly drops (see Figs. 7 and 8).

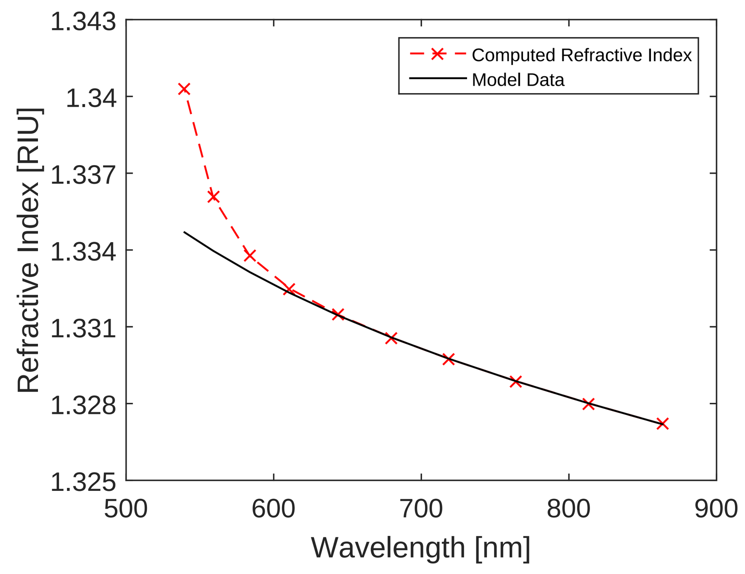


**Figure 7:** *Refractive index of water as a function of wavelength (the first procedure).*

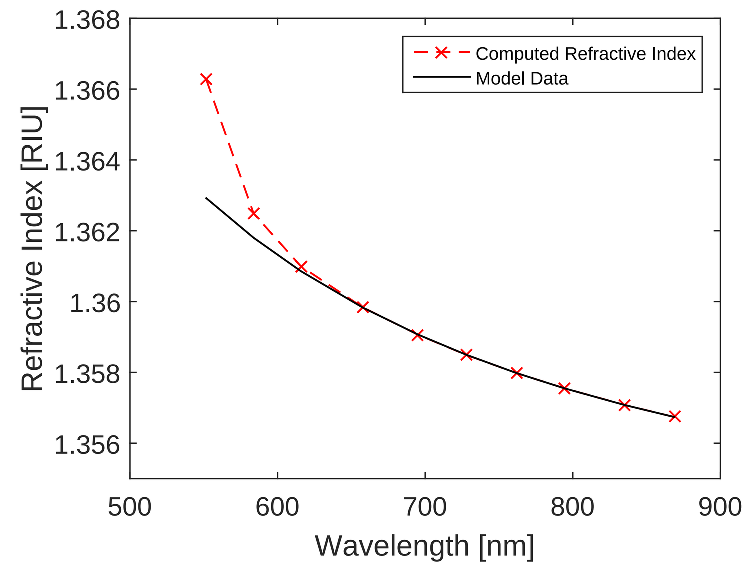


**Figure 8:** *Refractive index of ethanol as a function of wavelength (the first procedure).*

The second procedure utilizes the dependence of the reflectance ratio on the wavelength for different refractive indices. More specifically, an interval of wavelengths around the resonance wavelength is used to create minimum (dip) for each refractive index from the considered interval. This iterative method is based on finding the best match of the resonance wavelength to extract a value of the refractive index. This method has proven to be time efficient and more accurate than the first procedure (see Figs. 9 and 10).

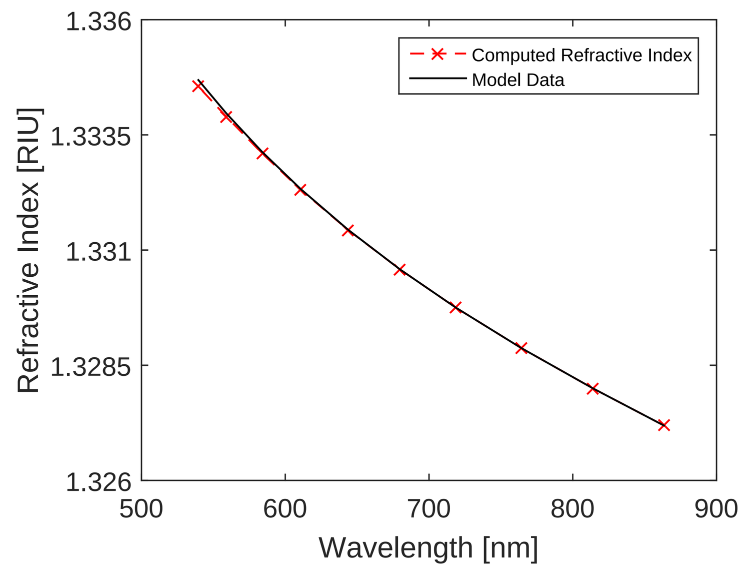


**Figure 9:** *Refractive index of water as a function of wavelength (the second procedure).*

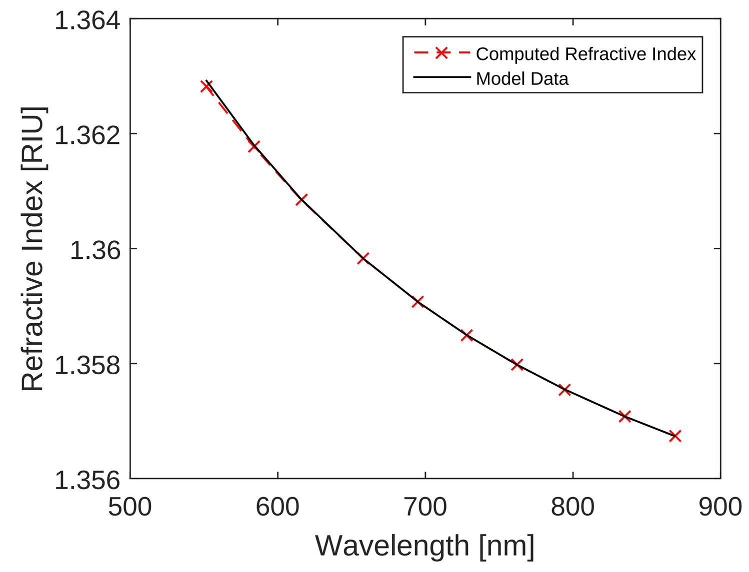


**Figure 10:** *Refractive index of ethanol as a function of wavelength (the second procedure).*

Results obtained using the second procedure for both analytes are specified in Tabs. 1 and 2. In these tables, $n$ is the refractive index computed by this method and $\Delta n$ represents the difference between the value of computed refractive index and the model value.

| $\theta$ [deg] | $\alpha$ [deg] | $\lambda_r$ [nm] | $n$ [RIU] | $\Delta n$ [RIU] |
|---|---|---|---|---|
| 53.0 | 12.02 | 863.41 | 1.3271920 | 0.00000017 |
| 53.3 | 11.51 | 813.61 | 1.3280050 | 0.00000003 |
| 53.7 | 10.84 | 736.83 | 1.3288800 | 0.00000011 |
| 54.2 | 9.99 | 718.68 | 1.3297505 | 0.00000008 |
| 54.8 | 8.96 | 679.94 | 1.3305775 | 0.00000041 |
| 55.6 | 7.59 | 643.89 | 1.3314320 | 0.00000492 |
| 56.7 | 5.70 | 610.88 | 1.3314320 | 0.00001323 |
| 58.1 | 3.29 | 583.82 | 1.3331025 | 0.00003211 |
| 60.2 | -0.35 | 558.95 | 1.3338980 | 0.00007153 |
| 63.0 | -5.21 | 539.56 | 1.3345500 | 0.00014060 |

**Table 1:** *Table specifying the results obtained using the second procedure (water).*

| $\theta$[deg] | $\alpha$[deg] | $\lambda_r$[nm] | $n$ [RIU] | $\Delta n$ [RIU] |
|---|---|---|---|---|
| 54.8 | 8.91 | 868.83 | 1.3567390 | 0.00000020 |
| 55.0 | 8.57 | 834.86 | 1.3570815 | 0.00000003 |
| 55.3 | 8.06 | 794.03 | 1.3575530 | 0.00000073 |
| 55.6 | 7.55 | 761.75 | 1.3579825 | 0.00000037 |
| 56.0 | 6.87 | 727.79 | 1.3584980 | 0.00000038 |
| 56.5 | 6.01 | 695.26 | 1.3590665 | 0.00000057 |
| 57.3 | 4.65 | 657.66 | 1.3598295 | 0.00000228 |
| 58.7 | 2.24 | 615.68 | 1.3608525 | 0.00000925 |
| 60.5 | -0.86 | 583.76 | 1.3617685 | 0.00003324 |
| 63.9 | -6.77 | 551.70 | 1.3628125 | 0.00010684 |

**Table 2:** *Table specifying the results obtained using the second procedure (ethanol).*

## 3. Conclusions

- Theoretical model of the Kretschmann configuration based refractometer was utilized to generate the spectral dependence of the reflectance ratio $R_p(\lambda)/R_s(\lambda)$ from which the refractive index at a specific resonance wavelength was computed.
- Theoretical results of retrieving the information about the dispersion of an analyte using two procedures were presented.
- Results obtained by the second procedure are in very good agreement with model representation of the dispersion of the analyte and the precision of the determination of computed values is very high despite it is decreasing with greater angles of incidence.

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## References

[1] P. Hlubina *et al*, *Opt. Commun.* **354**, 240–245 (2015).
[2] P. Hlubina, D. Ciprian, *Plasmonics* **12**, 1071–1078 (2017).